MONTHLY NOTICES

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Professor R. A. Sampson, M.A., D.Sc., F.R.S, PRESIDENT, in the Chair.

Thomas Hugh Close, Chief Draftsman, Existing Lines Department, Sydney, Romney, Fern Street, Pymble, Sydney, Australia;

Charles Monro Hector, M.D., B.Sc., Lower Hutt, Wellington, New Zealand;

Robert Meldrum Stewart, Superintendent, Time Service, The Observatory, Ottawa, Canada;

S. D. Tscherny, Professor in the University and Director of the Observatory, Warsaw, Russia; and

Charles J. Westland, Glenmuick, Christchurch, New Zealand,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Thomas Ralph Merton, B.Sc. (Oxon.), Lecturer in Spectroscopy, King's College, London (proposed by J. W. Nicholson).

Professor Solon I. Bailey, Harvard College Observatory, Cambridge, Mass., U.S.A.;
M. Charles Fabry, Professor of Physics in the University, Marseilles; and
Dr. W. H. Wright, Astronomer at the Lick Observatory, Mount Hamilton, California, U.S.A.,

were proposed by the Council as Associates of the Society.

Fifty presents were announced as having been received since the last Meeting, and the thanks of the Society were returned to the respective donors.

Proposal for a new Method of Determining the Constant of Aberration. By W. de Sitter, Ph.D.

1. Let four stars, or fields of stars, be selected in the equator at intervals of six hours. Let them be called I., II., III., IV., and let their right ascensions be α_1, α_2, α_3, α_4. At the epoch when I. is in the meridian at midnight the two arcs I.–IV. and II.–I. can be measured; the first or western angle (W) before midnight, and the eastern angle (O) after midnight. The difference of the two angles will be affected by aberration. Neglecting for the moment the influence of refraction, proper motion, etc., we have:

$$\begin{aligned}
&\text{At epoch I.}: (W-O)_1 = (\text{I.–IV.}) - (\text{II.–I.}) + K_1.\kappa,\\
&\text{,, ,, II.}: (W-O)_2 = (\text{II.–I.}) - (\text{III.–II.}) + K_2.\kappa,\\
&\text{,, ,, III.}: (W-O)_3 = (\text{III.–II.}) - (\text{IV.–III.}) + K_3.\kappa,\\
&\text{,, ,, IV.}: (W-O)_4 = (\text{IV.–III.}) - (\text{I.–IV.}) + K_4.\kappa,
\end{aligned}$$

where κ is the constant of aberration. Adding, we find:

$$\Sigma(W-O) = \Sigma K.\kappa.$$

Calling now A_1 and D_1 the right ascension and declination of the point on the ecliptic whose longitude is 90° behind the sun at the epoch I., we have

$$K_1 = 2\cos D_1\left[\sin(A_1-\alpha_1) - \cos\frac{\alpha_4-\alpha_2}{2}\sin\left(A_1+\frac{\alpha_4+\alpha_2}{2}\right)\right].$$

The ideal case is $\alpha_4-\alpha_2 = 180°$, $\alpha_4+\alpha_2 = 2\alpha_1$, and $A_1-\alpha_1 = 90°$. If these conditions were exactly fulfilled the factor within the square brackets would be unity. For differences of 1° in the arc $\alpha_4-\alpha_2$ and a fortnight in the epoch the factor is only diminished to 0·96. We can thus on the average take $K = 1\cdot96\cos D$. Now neglecting $\sin^6\epsilon$, we have

$$\Sigma\cos D = 4 - \sin^2\epsilon - \tfrac{1}{4}\sin^4\epsilon(1+\tfrac{3}{2}\sin^2 2\alpha),$$

where α is any of the right ascensions $\alpha_1 \ldots \alpha_4$. The effect of the choice of the areas is thus very small: $2\Sigma \cos D$ varies between 7·65 and 7·67. We can thus reasonably expect a factor

$$\Sigma K = 7{\cdot}5.$$

2. *Parallax* is rigorously eliminated if we observe at the exact epochs, and only a negligible effect can remain if the actual epochs are not exactly the best ones. There is thus no objection to the use of bright stars with unknown parallaxes.

Proper motion is not eliminated. We easily find

$$\Sigma(W - O) = \mu_4 - \mu_1.$$

To eliminate the effect of proper motion we must combine four series,* one beginning at epoch I., the next at epoch II., and so on, the observations being continued throughout seven epochs, or one year and a half. In other words, the several epochs must not have the same weight, but the relative weights must be:

At Epoch	I. . .	. weight	1
,, ,,	II. . .	,,	2
,, ,,	III. . .	,,	3
,, ,,	IV. . .	,,	4
,, ,,	V. = I. + 1 year	,,	3
,, ,,	VI. = II. + 1 ,,	,,	2
,, ,,	VII. = III. + 1 ,,	,,	1

Other corrections—nutation, diurnal aberration, aberration due to the eccentricity and the perturbations of the earth's orbit—should of course be applied. Refraction will be treated separately below.

3. The principal advantage claimed for the present method is its strictly differential character. The whole determination rests on the difference of two nearly equal arcs, which are measured on the same night with a few hours' interval. To fix ideas, let us suppose the observations to be made by Professor Turner's device for measuring arcs of 90°, explained in *M.N.*, vol. lxxi. Referring to fig. 9 on p. 428 of the volume quoted, we see that as the points

* We might derive the difference of proper motions $\mu_4 - \mu_1$ from the comparison of the arc I.–IV. as measured at the two epochs I. and IV. But we would then have to rely on the constancy of the instrumental arc of 90° for nine months, and the differential character of the method would be lost. Moreover, the loss of weight would be not inconsiderable. The mean error of $\mu_4 - \mu_1$ thus determined would be $\pm\frac{4}{3}\sqrt{2}.\epsilon$ (for the meaning of ϵ, see art. 4 below), and the resulting mean error of κ from one series would be increased from $\pm 0{\cdot}370\,\epsilon$ to $\pm 0{\cdot}435\,\epsilon$.

The constancy of the angle can, of course, be verified by Turner's cube (*M.N.*, lxxi. p. 428, fig. 8). It must be left to practical considerations to weigh the disadvantages connected with this introduction of another instrumental adjustment, and with the loss of weight, against the difficulty of securing the necessary number of observations at the middle epochs, which must have the larger weights.

S_1 and S_2 in their daily motion describe the equator from east to west, the points T_1 and T_2 describe the horizon from west to east, always conserving their distance of 90°. The mirror M remains fixed. The stability of this mirror is, however, only of secondary importance. A slight displacement would throw the stars out of the field, but would have no effect on the measured arc. The only quantity which must be trusted to remain constant during one night is the angle between the collimation lines of the two telescopes. This angle need not be known; it need not be exactly 90° [indeed it may be advantageous to make it 89° 58′ to allow for refraction]; it must only be constant. It would, of course, be possible to measure the arcs of 90° by any other instrument, such as M. Bigourdan's "*Comparateur*," described in *Comptes Rendus* for 1915 January 25, which is just coming to

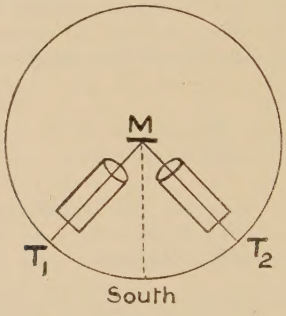


FIG. 1.

hand as this paper is being prepared for the press. But Turner's method seems to give the best chances of stability.

The invariability of the arc $T_1 T_2$ is of course best secured if the two telescopes are permanently fixed in azimuths + and − 45°, as shown in the figure.

But suppose them to be moved in azimuth without changing their angle, then we find the following zenith distances ζ_1, ζ_2, and angles of incidence N_1, N_2 for the two stars S_1, S_2 for different hour angles τ of the middle point (the table has been computed for latitude $\phi = 50°$):

τ	ζ_1	ζ_2	N_1	N_2
h m	°	°	°	°
0 0	63·0	63·0	48·4	48·4
0 30	67·0	59·3	55·1	41·8
1 0	71·3	56·2	62·0	35·5
1 30	75·8	53·6	68·9	29·8
2 0	80·4	51·6	75·9	24·8

Thus if we wish to avoid large zenith distances, and large angles of incidence, we are restricted to small hour angles τ. The

interval between the two observations on the same night is 6 hours $- 2\tau$. On the whole, it seems best to take $\tau = 0$.

4. Let the mean error of one measured arc of 90° be ϵ. Then, taking one observation at each epoch, the mean error of $\Sigma(W - O)$ is $\pm 2\sqrt{2}.\epsilon$. Thus from one series the mean error of κ would be, taking no account of the effect of proper motion, $\pm \frac{2\sqrt{2}}{7\cdot5}\epsilon$, and from four series so combined as to eliminate proper motion, one half of this, or $\pm 0\cdot185\epsilon$.

The only other method hitherto used for the determination of the constant of aberration, which also depends on differences between two observations on the same night, is Küstner's [*Astr. Nachr.*, Band 126]. In Küstner's method we measure differences of zenith distances. If we take four stars (or groups of stars), the observations are made at four epochs, and two observations on each night, with intervals of six hours. Thus if we take one observation of zenith distance as equivalent to one measure of an arc of 90°, the number of observations for the two methods is exactly the same. In Küstner's method we must trust to the stability of the instrumental zenith, which is controlled by observations of the spirit-level. It is evident that the uncertainty introduced thereby is of quite a different order from the possible want of constancy of the angle between two fixed horizontal telescopes.

Further, in Küstner's method proper motion is not eliminated. The method here proposed is thus differential in a much higher degree than Küstner's.

Moreover, the factor is about twice as large. For Küstner's method, taking four stars of declination $\delta = \phi$, we find for the factor of κ in one difference of zenith distance,

$$K = 2 \sin\phi \cos D \sin\frac{\alpha_1 - \alpha_2}{2} \sin\left(A - \frac{\alpha_1 + \alpha_2}{2}\right).$$

Taking the exact epoch $A - \frac{\alpha_1 + \alpha_2}{2} = 90°$, and also $\alpha_1 - \alpha_2 = 90°$, we find, for $\phi = 50°$,

$$K = 1\cdot083 \cos D,$$

as against $K = 2 \cos D$ for the method here proposed. Therefore, other things being equal [which they are not], with Küstner's method we require four times the number of observations to get the same mean error in the result.

Küstner's method, on the other hand, has the advantage of working near the zenith, and thus avoiding refraction. It is thus important to consider whether the advantages of the present method are not neutralised or outweighed by the uncertainty introduced by the correction for refraction.

5. *Refraction.*—The effect of refraction on the right ascension of an equatorial star is $k \tan t$, where t is the hour angle. We have,

for the two stars making an angle of 90°, $t_1 + t_2 = 90°$, $t_1 - t_2 = 2\tau$. Therefore the effect on the measured arc W or O is

$$2k \sec 2\tau.$$

The two arcs W and O are measured at the same hour angle τ. The effect of refraction on the difference W − O is therefore

$$2(k_w - k_o) \sec 2\tau.$$

As has been explained above, the best plan is to have $\tau = 0$, $\sec 2\tau = 1$. The difference $k_w - k_o$ depends on the temperature and the barometric pressure. Approximately we have, if the temperature is measured in degrees Centigrade and the pressure in millimetres:

$$2(k_w - k_o) = 0''{\cdot}43(T_o - T_w) + 0''{\cdot}15(B_w - B_o).$$

It is difficult to estimate the accuracy of the difference in refraction derived by the usual formulas from the readings of the barometer and the thermometer. Suppose we determine the difference of temperature $T_w - T_o$ with a mean error of $\pm 0°{\cdot}1$, and $B_w - B_o$ with a mean error of $0^{mm}.1$. The assumption made is not that we are able to read our meteorological instruments with this accuracy, but that the real difference of refraction is identical with the difference derived from our formulæ within limits of uncertainty corresponding to the stated mean errors of $T_w - T_o$ and $B_w - B_o$ respectively. Then the uncertainty in one measured value of W − O will be characterised by a mean error of $\pm 0''{\cdot}045$. If we take only *four* nights at each epoch, the corresponding mean error in $\Sigma(W - O)$ will again be $\pm 0''{\cdot}045$, and in $\kappa \pm 0''{\cdot}006$. If four series are combined so as to eliminate proper motion, the uncertainty in the resulting value of the constant of aberration due to refraction will be $\pm 0''{\cdot}003$.

6. *Alternative Method.*—If we measure azimuths instead of right ascensions, we get rid of refraction altogether.

We have $\tan A = \tan t \operatorname{cosec} \phi$, therefore, for $\phi = 50°$ and $t = 45°$, we have $A = 52°{\cdot}5$; the difference of azimuth of two consecutive areas when the middle point is on the meridian is therefore 105°. Now suppose two fixed horizontal telescopes making this angle symmetrically to the meridian, each with its own mirror.

The normals to the mirrors must be in the vertical planes passing through the optical axes of the telescopes, and inclined $45° - \frac{1}{2}\zeta$ to the horizon. For $\phi = 50°$ this becomes $13°{\cdot}5$. The angles of incidence are thus small. This is an advantage. Each area will be reflected by its own mirror into its own telescope. The mirrors must be fixed. The inclination of their normals is of secondary importance, but their orientation, *i.e.* the adjustment of their normals in the vertical planes through the optical axes of the telescopes, must be supposed to remain invariable during one night. This will not be difficult to realise.

The observations must be accurately timed, as much as possible

at the exact epoch $\tau = 0$. The observation is changed in character and becomes practically an observation of the (oblique) transits of the stars through the vertical planes of collimation of the two telescopes. It is evident that higher claims are made on instrumental stability than with the simple method of one mirror. Whether these disadvantages are outweighed by the advantage

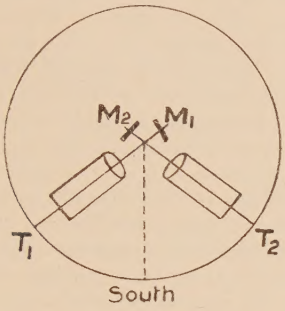


FIG. 2.

of the elimination of refraction must be left to experiment to decide.

Parallax and proper motion are eliminated as before. The factor now becomes (for the ideal case of arcs of exactly 90° and observations at the exact epochs):

$$K = 2 \operatorname{cosec} \zeta \sin (q - D),$$

where q is determined by

$$\sin q = \cos A \sec t, \qquad \text{or} \qquad \tan q = \tan \phi \operatorname{cosec} t.$$

In $\Sigma(W - O)$ we have thus

$$\Sigma K = 2 \operatorname{cosec} \zeta [\sin q \Sigma \cos D - \cos q \Sigma \sin D].$$

Now $\Sigma \sin D$ is zero, and consequently the factor for the original method is multiplied by $\sin q \operatorname{cosec} \zeta$. For $\phi = 50°$ this is 0·965. The factor thus becomes 7·14 instead of 7·5. If the epochs are not exact, we have, still taking arcs of exactly 90°,

$$K = 2 \operatorname{cosec} \zeta [\sin (A_1 - \alpha_1) \cos D_1 \sin q - \sin D_1 \cos q].$$

The loss of factor due to deviation of the epoch from the exact date is thus also of the same order as in the method of right ascensions.

7. The word "new" in the title of this note is not meant to imply the hypothesis that the idea of determining the constant of aberration from measures of large arcs, or even from differences of such arcs, has never occurred to anybody besides the present author. It is, of course, at least as old as 1871. But the cyclical arrangement of the four equatorial arcs has never been used and, so far as

known to me, never been expressly proposed for the determination of the constant of aberration.

Professor Turner, when discussing his scheme of subdivision of the equator by arcs of 90° for the purpose of determining fundamental right ascensions, necessarily was led to consider the question of the correction of the observations for aberration. Incidentally, he mentions the possibility of determining the value of the constant from the differences of adjacent arcs.* But nothing is said about the arrangement of the observations or the choice of the arcs, and it is not clear from the quoted passage whether Turner (from whose point of view the constant of aberration was not, of course, the primary object of investigation) did realise the full capacity of the method as a means of determining this constant.

The well-known method of Loewy, which was developed in a series of communications to the Académie des Sciences in the course of the year 1887 (*Comptes Rendus*, vol. civ.), depends on the comparison of *two* arcs, differently affected—preferably one of them not affected—by aberration, at two epochs. The method is thus very different from the one proposed above. Theoretically it is differential, in so far as the lengths of the arcs and the variation of the angle of the prism between the two epochs (not between the two observations at the same epoch, which are nearly simultaneous) are eliminated from the final result. But the completeness of this elimination must depend on the extent to which the instrument can be trusted to be invariable under the conditions in which the observations are made. The two arcs are observed in widely different azimuths. The optical and mechanical stability of the equatorial telescope, with its prism mounted before the object-glass, is naturally very different from that of a system consisting of two fixed horizontal telescopes and a fixed mirror. The fact that the two areas at both ends of one arc are viewed through different parts of the object-glass introduces a new source of errors. It may be noted that theoretically the maximum factor—for the same number of observations—is the same as in the method of the four equatorial arcs; but if day observations are to be avoided, it is not possible to observe at the exact epoch, and according to Loewy's estimate not more than $\frac{3}{4}$ of the maximum factor can be obtained in practice and not more than $\frac{1}{2}$ in an extended series of observations (*l.c.*, p. 1654). For the same reason proper motion can only very imperfectly be eliminated. Parallax is not eliminated at all. For all these reasons Loewy's method appears to me to be considerably inferior to the method proposed in the present paper.

* "Now we can compare any particular pair of arcs at times of year differing by a few months, . . . and in this way we may get a sufficiently strong determination of aberration which can be used for correcting the comparisons." *Monthly Notices*, lxxi. p. 434.

Leiden:
1915 *February.*

A Proposal for the Comparison of the Magnitude Scales of the Astrographic Catalogue. Sixth Note. The Oxford Magnitudes. With a Preliminary Discussion of the Existence of Obscured Patches in the Sky. By H. H. Turner, D.Sc., F.R.S., Savilian Professor. *With a letter from* F. G. Brown.

1. The previous notes are in *Monthly Notices*, lxix. p. 392, lxxii. p. 464 and p. 700, lxxv. p. 57 and p. 143. The proposal is

That the number of images recorded under each unit of the magnitude scale be counted and tabulated.

Complete zones of the Bordeaux, Algiers, Cape, and Perth (W.A.) portions have been given as examples, and two zones $+64^\circ$ and $+62^\circ$ of the Vatican results have been counted: but since these last counts are not very accordant, it is considered better to count yet another zone before presenting the figures. The counts for the Oxford zones will be given in full detail in vol. viii. of the Oxford Astrographic Catalogue, so that no extended tables need be given here: but it is desirable to have for comparison with other zones the figures of Table I.

2. In Table I. are given a few counts from the middle zone ($+28^\circ$) of the seven ($+25^\circ$ to $+31^\circ$) which form the Oxford share. The numbers of plates in 0^h, 1^h, and 2^h are 7, 7, 6, and this triplet repeats for the other octants of R.A. It has not been considered necessary here to reduce these figures to uniformity. The counts have actually been made to every unit of recorded diameter (d) of the star image: a selection is made here for brevity, but the counts for the fainter stars are given in some detail, in order to exhibit the vagaries of the scale near the limit of faintness —partly due to idiosyncrasies of the measurers, which will be fully investigated. But here as elsewhere it is considered that the total number of stars on the plate is steadily consistent. For a few plates the measurement was limited to the brighter stars to save labour: but the number of stars omitted was counted and printed in the volumes, and the requisite additions are made in the column "All."

[TABLE.

TABLE I.

Oxford Counts for Zone + 28°.

R.A.	50 and over.	49 to 40.	39 to 30.	29 to 18.	17 to 12.	11 to 9.	8 and 7.	6 and 5.	4 and 3.	Not Measured.
0h	11	21	55	258	402	667	220	208	57	
1	13	22	54	236	291	599	240	243	0	
2	23	21	61	180	225	289	263	374	22	
3	27	35	70	215	350	340	390	575	0	
4	13	20	33	119	206	163	200	327	18	
5	16	25	69	270	462	596	546	895	274	
6	12	27	86	510	1133	1643	1091	1772	649	
7	13	15	37	180	546	694	868	1244	416	
8	10	25	70	354	537	655	285	176	0	
9	9	22	57	434	748	1261	340	48	0	
10	8	9	61	260	354	596	244	149	0	
11	13	22	57	203	368	492	129	102	0	
12	17	22	48	317	448	958	244	12	0	
13	18	22	53	255	448	624	149	4	0	
14	9	16	42	166	202	405	249	67	16	
15	10	17	43	222	333	606	165	63	15	
16	4	22	58	310	346	907	197	0	0	
17	13	24	83	312	473	946	146	1	0	
18	43	103	263	1296	977	1024	286	445	384	2415
19	16	43	153	774	1257	2475	1416	683	564	518
20	20	43	108	626	921	1189	306	201	110	1400
21	11	20	75	385	853	1145	893	1039	439	
22	13	16	63	290	734	959	518	401	183	
23	8	17	34	173	287	477	398	154	27	
Totals	350	629	1733	8345	12901	19710	9783	9183	3174	4333
N′	350	979	2712	11057	23958	43668	53451	62634	65808	70141
log. N_0'	0·340	0·787	1·229	1·840	2·176	2·436	2·524	2·596	2·614	2·642
Mag. C. and M.	7·71	8·62	9·58	11·03	11·90	12·62	12·88	13·12	13·16	13·23

3. At the foot of the column is given the total number of stars for the zone and then the cumulative totals N′. But we may give these totals rather more fully as below. N′ being the cumulative total, N_0' is the average per plate; and since there are 160 plates in the zone

$$\log N_0' = \log N_0 - 2{\cdot}204.$$

The magnitude on the scale of Chapman and Melotte is deduced from log N_0' by use of the table on p. 145 of this volume. This is compared with the "observed" magnitude deduced from the formula

$$m = 16{\cdot}00 - 1{\cdot}18\sqrt{d}$$

found empirically. In the printed volume of Oxford measures for zone + 28° it will be seen that two formulæ were in use—

$a - 1{\cdot}02\sqrt{d}$ for plates taken earlier than No. 1558

$a - 1{\cdot}25\sqrt{d}$ for plates taken later than No. 1558.

The need for the change seemed to come with the erection of a new dome in 1900, with a much wider shutter which may have allowed wind to affect the telescope more (see *Mon. Not.*, lxv. p. 769). A fuller discussion of the change is reserved: it need only be remarked here that the coefficient $1{\cdot}18\sqrt{d}$ falls naturally between the two 1·02 and 1·25.

TABLE II.

Measured Diameters compared with Chapman and Melotte's Scale.

d.	Log N_0'.	Mag. C. and M.	Mag. observed.	O − C. and M.
50	0·340	7·71	7·66	− ·05
45	0·516	8·06	8·08	+ ·02
40	0·787	8·62	8·53	− ·09
35	0·962	8·99	9·01	+ ·02
30	1·229	9·58	9·53	− ·05
26	1·376	9·91	9·98	+ ·07
22	1·586	10·40	10·47	+ ·07
18	1·840	11·03	11·00	− ·03
15	1·946	11·29	11·43	+ ·14
12	2·176	11·90	11·92	+ ·02
9	2·436	12·62	12·46	− ·16
7	2·524	12·88	12·87	− ·01
5	2·596	13·12	13·36	+ ·24
3	2·614	13·16	13·96	+ ·80
All	2·642	13·23	...	...

4. The scale may be regarded as satisfactorily uniform until just near the end. If all stars fainter than $d = 7$ had been called 6, for which the formula gives $m = 13{\cdot}11$, the scale would have been as good as could be expected. The measurers apparently discriminated more than was necessary. Does this suggest that discrimination is easier when the images are near the limit of photographic faintness? Or is it merely that the range of error is greater? I do not feel sure that this point has been fully examined.

5. The main point of the present note is to draw attention to the excess or defect of faint stars in different parts of the Oxford zones. In dealing with other zones, a limited number of points in the scale of magnitude have been selected, and we shall probably select at least the following points

d =	40	30	18	9	All
mag. =	8·6	9·6	11·0	12·6	13·2

for complete discussion of the Oxford zones. But for the present it will suffice to confine attention to two points, viz.

$d = 26$ (mag. 9·9) and "All" (mag. = 13·2),

in order that a comparison may be shown more readily between several different zones. Accordingly, in Table III. are given the differences of log N for these two points (diminished by the constant shown at the foot of each column) for each of the zones $+26°$, $+28°$, $+29°$, $+30°$, and $+31°$. (Zones $+25°$ and $+27°$ have not yet been counted.)

6. The means given at the feet of the columns show sensible differences. One contributory factor is that although one end of the interval measured remains (it is hoped) fairly constant, the end represented by $d = 26$ does not always mean the same thing. It has been already stated in § 3 that two different formulæ were found necessary. A more accurate survey is being made, but the present is sufficient to show the main facts.

TABLE III.

Values of log N_{All} − log N_{26} for different Oxford Zones.

(The unit is 0·01.)

Decl. / R.A.	+26°.	+28°.	+29°.	+30°.	+31°.	Sum.	Gal. Lat. of +29°.
0h	+11	− 8	−12	− 1	+19	+ 9	−34°
1	+25	−13	−16	+ 3	+ 5	+ 4	−33
2	+ 9	−25	− 5	− 2	+ 4	−19	−28
3	−15	−20	− 3	− 8	− 9	−55	−21
4	−13	−19	−10	+ 6	+ 3	−33	−12
5	−15	+10	− 9	− 6	+ 3	−17	− 1
6	− 3	+30	+13	+ 5	−14	+31	+10
7	− 8	+37	+27	+ 9	− 5	+60	+22
8	− 3	− 8	− 4	− 2	− 9	−26	+35
9	−15	+14	−10	−17	−15	−43	+48
10	+ 2	− 5	−10	− 4	− 8	−25	+61
11	+ 1	− 7	−36	−13	− 3	−58	+74
12	−28	+ 3	− 7	− 8	−13	−53	+87
13	−12	−12	− 2	−23	− 6	−55	+78
14	−19	−12	+20	− 9	0	−20	+66
15	− 4	− 2	− 4	− 8	− 7	−25	+53
16	+18	− 5	− 5	+ 6	+ 6	+20	+40
17	+25	−10	+ 4	+13	+ 1	+33	+28
18	+17	−14	+13	+ 9	+22	+47	+15
19	− 1	+16	+ 1	+10	+21	+47	+ 3
20	+ 9	+ 8	+12	− 4	+ 8	+33	− 8
21	+ 9	+24	+ 1	+ 9	+ 9	+52	−18
22	+13	+16	+ 8	+ 7	+11	+55	−26
23	− 9	+ 7	+14	+30	+ 2	+44	−31
Mean	106	124	122	107	106		

7. It will be seen from the column of sums for all four zones that the circuit is divisible into four portions, viz.—

Portion I. from 16^h through 0^h to 1^h inclusive, where the differences are positive.
Portion II. from 2^h to 5^h, where they are negative.
Portion III. from 6^h to 7^h, again positive.
Portion IV. from 8^h to 15^h, again negative.

The different zones show some variations: thus for zones $+28°, +29°$, and $+30°$ Portion I. stops at 23^h: and these three zones are again entirely responsible for the positive value of $6^h - 7^h$, which does not occur in the other zones. Are these differences real or accidental? The narrowness of the zones in declination, and the actual overlap, suggests that they must be regarded as accidental. To test the point fully, however, we can make the further examination in two ways:

(α) Treating the differences as possibly real, we can group together all those cases where the differences are of much the same value, wherever they occur in the zones.

(β) Treating them as accidental, we can group all five zones in the manner indicated above, suggested by the sums of all five zones.

8. First we will take method (α). The material was grouped simply according to the value of the hourly group shown in Table III:

(A) all values exceeding $+9$ (26 hourly groups)
(B) ,, from $+1$ to $+9$ (29 ,,)
(C) ,, ,, 0 to -9 (40 ,,)
(D) ,, less than -9 (25 ,,)

The use of log N or N was now abandoned in favour of the original counts, which apply to stars between narrow limits of magnitude, and are not cumulative or integrated counts to a limit. To avoid undue influence of densely populated regions, the counts were divided (or sometimes multiplied) by some simple number so as to bring the total number of stars in an hourly group to about 2000–3000: and on adding together these modified counts the results shown in Table IV. were obtained.

TABLE IV.

Counts of Stars in Oxford Zones for Groups A, B, C, D.

d.	>79.	>59.	>49.	>39.	>29.	>21.	>14.	>8.	>0.
Limiting Mag.	5·3.	6·9.	7·7.	8·6.	9·6.	10·4.	11·3.	12·6.	13·0.
A	9	58	139	386	1146	2645	6391	23719	19774
B	27	81	185	605	1717	3606	8655	27363	11617
C	45	189	306	975	2779	5626	11948	39047	12011
D	41	137	268	732	1886	3624	7407	21672	9464
Totals	122	465	898	2698	7528	15501	34401	111801	52866

9. The top line shows the limiting values of the diameter, the counts referring to diameters greater than the limit and less than the limit for the column to the left. The limiting magnitude on C. and M. scale was derived both from Table II., which refers to a single zone ($+28°$) only, and from the totals shown at the foot of Table IV., which refer to all zones but give very different relative importance to the Galaxy. A surprisingly close accordance was found, but as this is not of immediate importance the details need not be given here. The limiting magnitude given in the second row of the table shows sufficient for our purpose.

10. We now take logarithms of the rows A, B, C, D: we then subtract log A from the logs of B, C, D to form log ratios: finally, adopting the column headed >39 or mag. 8·6 as standard, we subtract the values of log B/A, log C/A, log D/A for this column from all members in the same row. We thus arrive at the log ratios of Table V., expressing the residuals in units of 0·001 for convenience.

TABLE V.

Log Ratios of Table IV. in Units of 0·001.

Mag.	5·3.	6·9.	7·7.	8·6.	9·6.	10·4.	11·3.	12·6.	13·0.
log B/A	+282	− 49	−71	0	−19	− 60	− 64	−133	−426
log C/A	+297	+112	−59	0	−17	− 74	−131	−186	−618
log D/A	+381	+ 96	+ 7	0	−61	−141	−215	−317	−598

11. The striking features of Table V. are the rapid falls at the ends of the rows, both for very bright and very faint stars. Now it is well known that on nights when the images are unsteady, either from actual shaking of the telescope by wind or atmospheric tremors, the faintest images are apt to be lost altogether, because an image is not long enough focussed on one particular spot. For the brightest stars the images are enlarged in diameter; and though they are also fainter, measures of diameter take account only of the enlargement and not of the consequent faintness. Hence the stars appear unduly bright. We conclude that our method of selecting the groups A, B, C, D has put nearly all the unsteady nights into groups B, C, D, so that they show, by comparison with A, both the above features conspicuously. If, on the other hand, we group the same material as suggested in § 7 from inspection of the last column of Table III., we shall distribute the unsteady nights more equably among the four groups, leaving actual stellar features a better chance to reveal themselves.

12. It seems unnecessary to give the actual counts of the rearranged material. We may proceed at once to the log ratios corresponding to Table V., and these are given in Table VI.

TABLE VI.

Log Ratios for the Groups specified in § 7.

Mag.	5·3.	6·9.	7·7.	8·6.	9·6.	10·4.	11·3.	12·6.	13·0.
log II./I.	+232	+ 63	+15	0	−83	−128	−149	−120	−321
log III./I.	+164	− 28	−15	0	+15	+ 39	+ 8	+ 27	− 71
log IV./I.	+103	+ 86	+27	0	−16	− 16	− 66	− 55	−487
log II./III.	+ 68	+ 91	+30	0	−98	−167	−157	−147	−250
log IV./III.	− 61	+114	+42	0	−31	− 55	− 74	− 82	−416

13. Comparing Table VI. with Table V. we see that the features which may be ascribed to accidental causes, if not perhaps entirely eliminated, are at any rate largely reduced. Portion III. does not differ essentially from Portion I. It will now be provisionally assumed that Portions I. and III. are unobstructed, but that there is some obstructing cloud of matter in the directions included in Portions II. and IV. Evidence in favour of such a hypothesis has been accumulating, as has been indicated in previous papers of this series, and though we cannot hope to get at the complete facts without a more systematic discussion, we may venture on a rude first approximation.

14. It is scarcely necessary to repeat reasons for dissociating the features under discussion from any effect of the Galaxy. The galactic latitude for zone +29°, which is approximately the mean zone, is given in the last column of Table III. Portions I. and III. do indeed seem associated with the Galaxy at first sight: but if the effect were galactic, 4^h and 5^h should (in the seventh column of Table III.) be positive, whereas there is a striking difference between them and 7^h. Again, the great extent (16^h–1^h) of Group I., at a nearly constant value, though the galactic latitude changes considerably, argues against a galactic effect. At the same time it may be stated here that, *when the "obscuration" effect discussed below is eliminated*, a small galactic effect is revealed. The definitive discussion of this effect is reserved.

15. Portion III. is specially valuable as showing that the effect is not seasonal in some way—due perhaps to variation in conditions of exposure throughout the year. We cannot explain in that way the very definite interruption between Portions II. and IV. In the last two lines of Table VI. the obscured portions are compared with Portion III., and the log ratios show a smoother curve than those with Portion I. The drop for the bright stars is much less marked indeed for log IV./III.; it is difficult to say whether there is any real drop before magnitude 8·6. The scarcity of bright stars makes the accidental errors large in this part of the curve. It is perhaps better to combine the first two groups into one. For a different reason we shall do well to combine the last two groups into one: there are, of course, plenty of stars in these groups to eliminate purely accidental errors, but experience shows (and

several illustrations of the fact have been already given in these papers) that observers are liable to *systematic* errors in assigning classes to the faintest stars on the plate (see, for instance, *M.N.*, lxxv. p. 60, § 7), though the total number measured might be correct. Making these combinations, and further adding together the counts for the "obscured" and the "unobscured" portions, we get

TABLE VII.

Limiting Mag.	6·9	7·7	8·6	9·6	10·4	11·3	13·0
Obscured	346	496	1452	3871	7807	16607	71454
Unobscured	241	402	1246	3657	7694	17784	93222
Log Ratio	+·157	+·092	+·067	+·025	+·006	−·029	−·116
Log N_O/N_U	+·157	+·117	+·084	+·046	+·023	−·006	−·085
Calculated	+·160	+·128	+·092	+·052	+·020	−·016	−·084

16. The counts here given are modified counts, as mentioned in § 8. The obscured portions are R.A. 2^h–5^h and 8^h–15^h: the unobscured 6^h and 7^h with 16^h–1^h. The "log ratio" is the simple difference of the logarithms of the numbers given above.

Below this is given log N_O/N_U. The symbol N is here used to signify a cumulative count, formed by adding the numbers successively. N_O refers to the obscured portions and N_U to the unobscured. The "calculated" numbers represent a linear dependence on the limiting magnitude m, namely,

$$\log N_O/N_U = -·0400\ (m - 10·9).$$

The possibility of a simple formula of this kind is distinctly in favour of the combination of Groups II. and IV. On the other hand, the differences in their behaviour shown in Table VI. are rather too large to be treated as accidental without some further proof.

17. We now turn to other zones. Some indications of similar differences have already been given, but we will now put them into rather more definite shape. In each case we have at present only counted a single zone of 2° breadth in declination, instead of the five such zones available for Oxford. Hence we may perhaps omit the third decimal place in the logarithms henceforward. Moreover, the results must be treated as provisional until confirmed by counts of other zones. We have seen how in the case of the Oxford zones unsteady nights may unduly affect the results from one zone only.

The Bordeaux Counts.

18. The Bordeaux counts for zone +17° are given on p. 465 of *Monthly Notices*, vol. lxxii. On p. 467 it is shown that there is a considerable excess of bright stars in the non-galactic portion 8^h–17^h, though no such excess is shown in the other non-galactic portion 23^h–4^h. On p. 468 the coefficients of the first harmonic

of log N' are given for successive magnitudes as follows:—The first column gives the limiting magnitude on the Bordeaux adopted scale. Alongside this in the paper cited is given the magnitude on Kapteyn's visual scale; but we may now substitute that on the scale of Chapman and Melotte, as tabulated in *Monthly Notices*, lxxv. p. 145.

TABLE VIII.

First Harmonics of log N' for Bordeaux Zone + 17°.

Mag. Bord.	Mag. C. and M.	Coefficient of sin α.	Coefficient of cos α.	Equivalent.
6·75	7·73	−0·12	−0·19	0·22 cos (α − 213°)
7·75	8·86	−0·08	−0·17	0·19 cos (α − 205°)
8·75	10·11	−0·04	−0·13	0·14 cos (α − 198°)
9·75	11·29	−0·02	0·00	0·02 cos (α − 270°)
10·75	12·05	−0·03	+0·04	0·05 cos (α − 323°)
12·25	12·03	−0·03	+0·04	0·06 cos (α − 309°)

19. It is clear that there is a considerable change between magnitudes 10 and 11, which commences perhaps on the bright side of this limit. The maximum of bright stars assigned in this way is about 13^h, the middle of the portion 8^h–17^h above mentioned.

20. The Bordeaux zone was not treated in the manner adopted in recent papers: and we may rectify the omission by giving here Table IX., showing the differences of log N', referred to mag. 7·75 Bordeaux (= 8·86 C. and M.) as standard, these values being given in the second column. To the differences in each column must be added the constant at the foot.

TABLE IX.

Differences log N' − log N'_B for each Hour of R.A., in Units of 0·01.

R.A.	Mag. 8·86.	Mag. 10·79.	Mag. 12·05.	Mag. 12·63.	III. − II.	IV. + III. − II.
h						
0	146	+ 5	+ 7	+ 6	+ 2	+ 8
1	146	+ 1	+ 7	+ 4	+ 6	+ 10
2	132	+ 7	+ 15	+ 11	+ 8	+ 19
3	140	+ 6	+ 15	+ 16	+ 9	+ 25
4	140	0	− 5	− 4	− 5	− 9
5	173	+26	+ 27	+ 28	+ 1	+ 29
6	181	+25	+ 28	+ 33	+ 3	+ 36

TABLE IX.—*continued.*

R.A. h	Mag. 8·86.	Mag. 10·79.	Mag. 12·05.	Mag. 12·63.	III.–II.	IV.–III.–II.
7	210	– 1	– 9	– 8	– 8	– 16
8	193	– 7	– 13	– 18	– 6	– 24
9	191	+ 2	– 17	– 28	– 19	– 47
10	179	– 9	– 23	– 33	– 14	– 47
11	177	– 12	– 23	– 24	– 11	– 35
12	170	– 17	– 22	– 14	– 5	– 19
13	166	– 15	– 14	– 14	+ 1	– 13
14	174	– 11	– 26	– 30	– 15	– 45
15	166	– 9	– 9	– 2	0	– 2
16	170	– 24	– 18	– 4	+ 6	+ 2
17	218	– 14	– 23	– 27	– 9	– 36
18	198	+ 1	0	+ 1	– 1	0
19	206	+ 4	+ 5	+ 5	+ 1	+ 6
20	190	+ 6	+ 12	+ 13	+ 6	+ 19
21	149	+ 26	+ 36	+ 39	+ 10	+ 49
22	138	+ 14	+ 30	+ 33	+ 16	+ 49
23	143	+ 7	+ 17	+ 22	+ 10	+ 32
...	...	+ 84	+ 134	+ 154	+ 50	+ 204

21. It will be seen at a glance that the zone is divided into well-marked portions. From 7^h to 14^h the differences are negative, showing a defect of faint stars. From 20^h to 6^h the differences are distinctly positive. To show that this is not due either to (*a*) the uncertainties of enumerating the comparatively few bright stars shown in Column I., or again (*b*) to anomalies in the number of very faint stars which influence Column IV., the differences III.–II. have been formed, which largely mitigate both these objections. They show the division into positive and negative portions quite clearly.

22. The portion 15^h–19^h seems to be neither positive nor negative. Since this is close to the Galaxy (as indicated by the high numbers in the first column) we are clearly not in the presence of any phenomenon culminating in the Galaxy. At the opposite crossing we see that 7^h has the highest figure 210 in Column I., but belongs very distinctly to the negative portion, where faint stars are in defect, while the closely adjacent 6^h belongs just as clearly to the positive portion.

23. Having established this division, we may now return to the individual star counts and see where the faint stars begin to fail. The suggestion at present put forward is that the failure is due to obstruction of light of some kind, probably from finely

divided opaque matter scattered in space. If so, we may possibly find that the brighter stars are not affected. The tables above given scarcely suffice to tell us this, since all the brighter magnitudes have been taken together: and further, the cumulative process of forming N′ retains the peculiarities of bright stars to some extent among faint. Hence we return to the individual groups given in Table I. on p. 465 of *M.N.*, lxxii. This table we will divide into three portions:—

Portion I. From 20^h through 0^h to 6^h inclusive: rate of increase of faint stars definitely excessive.

Portion II. From 15^h to 19^h inclusive: rate nearly average.

Portion III. From 7^h to 14^h: rate definitely defective.

24. In Table X. is shown—

Firstly, the Bordeaux heading for the group, which may be taken to include stars one-quarter magnitude on each side. It was found that there were anomalies in the partition of groups 11·0, 11·5, and 12·0, but it is believed that these anomalies disappear in the sum of the three, which is given in the table.

Secondly, the *limiting* magnitude on the fainter side as shown by the scale of Chapman and Melotte, adapted for use as already mentioned. The Bordeaux scale is apparently about a whole magnitude in error.

Thirdly, the simple sums for each portion.

Fourthly, the logarithms, subtracting that for the second group from each row, so as to make the rows directly comparable, all signs positive, and the unit of 0·01 being used to avoid decimals.

Fifthly, the differences of these from Portion I.

Table X.

Bordeaux.

(Logarithms in Units of 0·01.)

Bord. Mag.	6·0.	6·5.	7·0.	7·5.	8·0.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0 to 12·0.
C. and M.	7·3.	7·7.	8·3.	8·9.	9·5.	10·1.	10·8.	11·3.	11·7.	12·1.	12·6.
I.	49	33	83	165	375	758	1820	2387	3255	2858	7771
II.	109	60	138	231	505	812	1632	1970	2754	2620	6734
III.	90	63	165	248	529	815	1317	1441	1975	1679	4306
II. + III.	199	123	303	479	1034	1637	2949	3411	4729	4299	11040
log I.	17	0	40	70	105	136	174	186	199	194	237
log II.	26	0	36	58	92	113	143	151	166	164	205
log III.	15	0	42	59	92	111	132	136	150	142	183
log (II. + III.)	21	0	39	59	92	112	138	144	158	154	195
log II./I.	+9	0	−4	−12	−13	−23	−31	−35	−33	−30	−32
log III./I.	−2	0	+2	−11	−13	−25	−42	−50	−49	−52	−54
log (II. + III.)/I.	+4	0	−1	−11	−13	−24	−36	−42	−41	−40	−42

25. The main point of the table is in the last three lines, which show that there is little difference between the first three columns

and again between the last four or five. The change takes place by successive steps in the intermediate three or four groups—say between magnitudes 8·5 and 10·5 on the adapted scale of Chapman and Melotte. Further, Group II. is clearly more nearly like Group III. than like Group I., and though the "obscuration" is apparently not quite so great in 15^h–20^h, we may at this stage neglect the difference and take the obscuration as extending from 7^h to 19^h inclusive. When counts from other zones become available we may discuss whether there are perhaps breaks in this long stretch similar to the Oxford break at 5^h–6^h.

26. There is just one further point to be examined in the Bordeaux counts. It will be seen in Table IX. that the numbers for 4^h R.A. are all negative. Were it not for the suggestion of other zones this single hour might pass as accidental, but since Oxford and Algiers have an "obscuration" in this neighbourhood, it seems well to examine the counts for 4^h. It will suffice to give the following:—

Mag.	8·3	9·5	10·8	11·7	12·6
4^h	8	45	121	229	388
(II. + III.)	625	1513	4586	8140	15339
log 4^h/(II. + III.)	0	+37	+32	+35	+30

There is thus no good case for considering 4^h as obscured. The number of bright stars (8) is clearly too small to accord with the other numbers; there should have been 18 rather than 8. But this must be treated as an accidental irregularity at present.

The Algiers Counts.

27. The separate counts are given in *Monthly Notices*, lxxii. p. 700. On p. 704 the first harmonics are calculated for each magnitude heading (*not* cumulative counts) as follows, the limiting magnitude on the adapted scale of Chapman and Melotte being now added to the table formerly given:—

TABLE XI.

First Harmonic for separate Groups of Algiers + 1°.

Algiers Mag.	C. and M. Limit.	First Harmonic as a Percentage.
Brter. than 7·0	7·83	$+34 \sin \alpha - 14 \cos \alpha = 37 \cos (\alpha - 113°)$
7·0–7·9	8·75	$+14 \sin \alpha - 22 \cos \alpha = 26 \cos (\alpha - 148°)$
8·0–8·9	10·35	$+35 \sin \alpha - 16 \cos \alpha = 38 \cos (\alpha - 114°)$
9·0–9·9	10·90	$+32 \sin \alpha - 10 \cos \alpha = 34 \cos (\alpha - 107°)$
10·0–10·9	11·81	$+29 \sin \alpha - 18 \cos \alpha = 34 \cos (\alpha - 122°)$
11·0–11·9	12·41	$+66 \sin \alpha - 11 \cos \alpha = 67 \cos (\alpha - 99°)$

28. The quantity tabulated here differs a little from that used elsewhere. The harmonic was calculated from the counts themselves instead of from their logarithms, and the coefficients were

then divided by the mean value and multiplied by 100 so as to give percentages. [In the former paper they were inadvertently printed with a decimal point, which gives them on a scale of 10 instead of 100 : but nothing turns upon their absolute magnitude.]

29. It will be seen that the coefficients are nearly the same, with the exception of the last, which differs considerably. On examining the separate figures it is found that on the evidence of the last group we can divide the circuit into various portions, viz.:—

Portion I. From 6^h to 9^h inclusive : rate of increase positive.
II. From 10^h to 18^h : rate negative.
III. From 19^h to 20^h : rate positive.
IV. From 21^h to 23^h : rate negative.
V. From 0^h to 3^h : rate positive.
VI. From 4^h to 5^h : rate negative.

These divisions are only preliminary, and to be tested by reference to the original counts. These are given in Table XII., with logarithms on the same plan as before, referred now to the *first* column as the smallest.

TABLE XII.

Algiers.

(Logarithms in units of 0·01.)

Algiers limiting Mag. C. and M.	6·9. 7·8.	7·9. 8·8.	8·9. 10·4.	9·9. 10·9.	10·9. 11·8.	All. 12·4.
I.	128	210	1854	1572	4791	7952
II.	135	285	1453	1145	3840	2628
III.	45	91	570	559	1703	2047
IV.	42	116	501	465	1216	1133
V.	33	34	439	365	1314	1316
VI.	68	85	482	426	995	1284
log I.	0	20	116	109	157	179
log II.	0	32	103	93	145	129
log III.	0	31	111	110	158	166
log IV.	0	44	108	105	146	143
log V.	0	1	112	104	160	160
log VI.	0	10	85	80	117	128
log II./I.	0	+12	−13	−16	−12	−50
log III./I.	0	+11	− 5	+ 1	+ 1	−13
log IV./I.	0	+24	− 8	− 4	−11	−36
log V./I.	0	−19	− 4	− 5	+ 3	−19
log VI./I.	0	−10	−31	−29	−40	−51

30. On the whole, the hypothesis that Portions II., IV., and VI. are obscured, while Portions I., III., and V. are not, is fairly justified, and we may try the effect of adding the two sets in order to diminish accidental errors. The result is shown in Table XIII.

TABLE XIII.

Algiers.

(Logarithms in units of 0·01.)

C. and M.	7·8.	8·8.	10·4.	10·9.	11·8.	12·4.
I., III., V.	206	335	2863	2496	7808	11315
II., IV., VI.	245	486	2436	2036	6051	5045
log (I., III., V.)	0	21	115	109	158	174
log (II., IV., VI.)	0	30	100	92	139	131
Diff.	0	+9	−15	−17	−19	−43

31. The obscuration does not begin until after magnitude 8·8, somewhat later than in the case of Bordeaux. The total amount is very much the same in the two cases, and in both distinctly greater than shown by the Oxford zones—nearly double the amount.

The Cape Counts.

32. The counts for $-41°$ are given in *Monthly Notices*, lxxv. p. 57. On p. 63 it is pointed out how definitely the circuit of R.A. divides into a negative or "obscured" portion 22^h–9^h and a positive or "unobscured" portion 10^h–21^h. The conditions are thus approximately the reverse of those in the northern hemisphere, where the portion 10^h–20^h is found "obscured" at Oxford, Bordeaux and Algiers, and the question arises how and where the transition occurs. The Perth ($-32°$) counts will throw some light on the question, but the woeful blank between Perth and Algiers leaves us in much doubt, as will presently be seen.

33. The counts already printed as above do not give details about the brighter stars, all measures of diameter greater than 2·00 on the Cape scale being taken together. Mr. Bellamy has kindly amplified the information by dividing this group into three. At the other end of the scale all the estimated stars are taken together, for the reasons given in § 7–10 of the paper cited. There seems no reason to subdivide the zone into more than two portions at present, though further subdivision may be attempted when other zones are available for counting.

TABLE XIV.

The Cape.

(Logs in units of 0·01; first term subtracted from each row.)

Limit of *d*.	3·00.	2·50.	2·00.	1·50.	1·30.	1·10.	1·00.	0·60.	All.
Mag. C. and M.	8·1.	8·7.	9·4.	10·0.	10·5.	11·1.	11·6.	12·2.	13·5.
10^h-21^h	196	216	434	755	1013	2532	3335	6354	28004
22^h-9^h	290	260	690	973	1016	2190	2326	4968	15867
log I.	0	4	35	59	72	111	123	151	216
log II.	0	−4	38	53	55	88	91	124	174
log II./I.	0	−8	+3	−6	−17	−23	−32	−27	−42

The drop does not begin decisively till about magnitude 10— perhaps a little earlier. The total amount is like that of Bordeaux and Algiers.

The Perth Counts.

34. The Perth counts are given in *Monthly Notices*, lxxv. p. 143. The table on p. 146 suggests provisional grouping in portions as follows:—

Portion I.	$13^h - 22^h$	rate of increase		positive
II.	$23^h - 3^h$	,,	,,	negative
III.	$4^h - 8^h$	,,	,,	positive
IV.	$9^h - 12^h$	,,	,,	negative

The counts for these portions at a limited number of points in the scale are as below:—

TABLE XV.

Perth (*W.A.*).

(Logs in units of 0·01; first term subtracted from each row.)

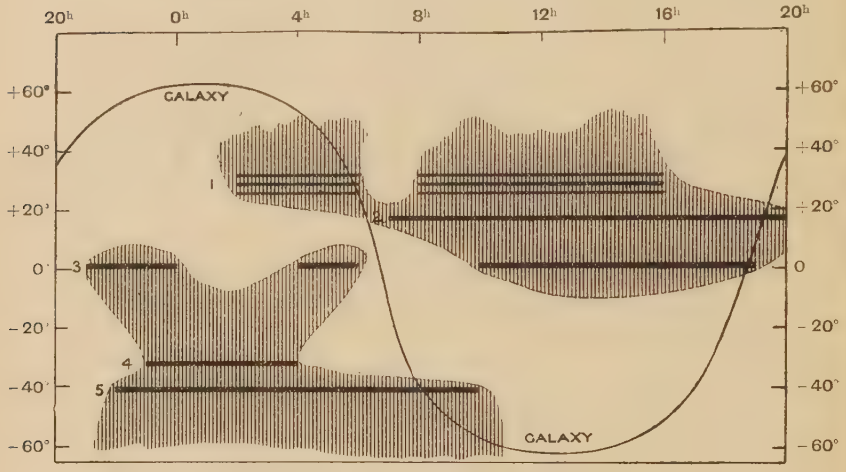
Perth Limit.	Brt.	B.	D.	F.	H.	J.	All.
C. and M.	7·8.	9·1.	10·1.	11·2.	12·3.	12·7.	13·1.
I.	294	836	2077	6000	14758	10007	14302
II.	62	214	454	1366	2163	1292	1049
III.	125	382	1059	3196	6953	4679	5863
IV.	92	341	806	2194	4782	2670	3721
log I.	0	45	85	131	170	153	169
log II.	0	54	87	135	154	132	123
log III.	0	48	93	140	174	157	167
log IV.	0	57	95	138	172	147	161
log II./I.	0	+ 9	+ 2	+ 4	− 16	− 21	− 46
log III./I.	0	+ 3	+ 8	+ 9	+ 4	+ 4	− 2
log IV./I.	0	+12	+10	+ 7	+ 2	− 6	− 8
I.+III.+IV.	511	1559	3942	11390	26493	17356	23886
log (sum)	0	48	89	135	171	153	167
log II./sum	0	+ 6	− 2	0	− 17	− 21	− 46

35. The log ratios show that there is no great difference between Portions I., III., and IV. Hence we may add them together as at the foot of the table: and it appears that the "obscuration" does not begin till after magnitude 11·2. It is confined to the small portion 23^h–3^h in R.A.

Summary.

36. The summary may be put in the form of a diagram. The thick black lines on the diagram represent the portions for which

there is positive evidence of obscuration of faint stars. The shaded portions are conjectural extensions, the main purpose of which is to illustrate the need of more material. The delay in publishing the measures of the Astrographic Catalogue is becoming tragic. If even only a single zone of San Fernando ($-3°$ to $-9°$), Tacubaya ($-10°$ to $-16°$), Hyderabad and La Plata ($-17°$ to $-23°$), or Cordoba ($-24°$ to $-31°$) were published, how much it



Obscured Patches in the Sky.
1. Oxford. 2. Bordeaux. 3. Algiers. 4. Perth. 5. Cape.

would tell us! But the Perth and Cape single zones are so far all that we have published of the southern hemisphere.

The obscuration is shown by the diagram in *position*. It remains to indicate its amount, and the magnitudes which are affected. The following table shows the approximate state of affairs:—

TABLE XVI.

Amount of Obscuration for Various Magnitudes.

(The unit is 0·01 of log ratio.)

Mag.	7·0.	8·5.	10·0.	11·5.	13·0.
Oxford + 29°	− 15	− 22	− 29	− 36	− 42
Bordeaux + 17°	0	− 24	− 42	− 42	− 42
Algiers + 1°	0	0	− 10	− 18	− 42
Perth − 32°	0	0	0	0	− 42
Cape − 41°	0	0	0	− 32	− 42

The final figures in all the zones except Oxford are close to those found above; they are all so close to − 42 that this final

value has been adopted. In the case of Oxford a constant has been subtracted from the numbers of Table VII. so as to make the terminal figure −42 for comparison with other zones. Where the obscuration begins, *i.e.* where the zero should be, is at present doubtful, but it must be earlier than 7·0. There is obviously something systematic about the above numbers.

As stated in § 14, when this "obscuration" effect is eliminated, a small galactic effect is revealed: but the discussion of this is reserved.

Appendix: Letter from Mr. F. G. Brown.

It will be seen that in the above discussion the use of harmonics has been abandoned in favour of a more direct statement. But the results might have been expressed by their use, taking a sufficient number into account: and one advantage of that form of expression is that comparison with other results is readily made. It will be remembered that three years ago Mr. F. G. Brown discussed the brightness of nebulæ in different parts of the sky, and the following interesting letter from him shows how confirmation of the above results is forthcoming from the nebular statistics. The fact stated at the end of the letter, that Mr. Brown was on his way to the Front, is, I hope, adequate reason for printing the letter as it stands. We may hope for a more extended examination on his safe return.

"Dear Sir,—May I say that I have read with much interest your two recent papers relating to the Comparison of Stellar Magnitude Scales, particularly your remarks at the end of the note on the Cape mags. as to the possible existence of a varying degree of absorption according to the direction in which we are looking.

"I thought it would be of interest to compare the harmonics on p. 65 based on the Cape plates with those derived from my figures for the brightness of the nebulæ and collected in Table I., p. 204, of your paper in *Monthly Notices*, January 1912, to which I have added the fourth harmonic. As there is no zone with centre at −41° in this table, I have taken the *weighted* mean of the zones of N.P.D. 90°–125° and 125°–150°. By the 'weighted' mean I mean that determined by the values of the coefficients. The phases are as follows:—

	I.	II.	III.	IV.	Reference.
A, Cape zone −41°	236°	261°	293°	309°	*M.N.*, Dec. 14, p. 65.
B, neb. zone 90°–125°	220	264	251	282	*M.N.*, Jan. 1912, p. 204, Table I.
C, ,, ,, 125°–150°	148	186	307	321	
D, weighted mean of B and C = zone −35°	200	249	288	295	
$\frac{A-D}{\text{Maximum possible}}$	·20	·13	·08	·31	

"The harmonics for the Perth plates (zone − 32°) also agree well.

	I.	II.	III.	IV.	Reference.
	°	°	°	°	
E, Perth zone, column H − D.	206	287	295	318	*M.N.*, Jan. 15.
$\frac{A-E}{\text{Maximum}}$	·03	·21	·12	·51	

"The arithmetical progression remarked on in your paper on the Cape mags. is in evidence in each case. The above results encouraged me to deal with the Bordeaux and Algiers zones in the same way. The residuals on which the harmonics were based were obtained in a similar manner to those in your Perth and Cape notes, and are as follows:—

Bordeaux.

Centre of R.A.	=15°	45°	75°	105°	135°	165°	195°	225°	255°	285°	315°	345°
Log N_0' to mag. 8·5	−32	−36	−8	+27	+13	−4	−22	−14	+13	+26	+5	−38
Residuals for mag. 12·0	+5	+9	+6	+4	−21	−24	−2	−9	−10	+2	+13	+28

Algiers.

Log N_0' to mag. 8·0	−62	−40	+16	+34	+3	+11	−11	−13	−13	+7	+11	−9
Residuals for mag. 12·0	+26	+16	−18	+24	+6	−30	−17	−9	−2	−11	+1	−25

"The harmonics obtained from these residuals, with those for the corresponding zones from the nebulæ, are as follows:—

Bordeaux +17°.

(Reference, *M.N.*, April 1912, p. 465.)

Bordeaux plates	$16\cdot3\cos(\theta-357^\circ)$	$+0\cdot8\cos 2(\theta-12^\circ)$	$+9\cdot4\cos 3(\theta-88^\circ)$	$+2\cdot5\cos 4(\theta-98^\circ)$
Neb. zone 55°−90°. Middle +17½°	$\cdot15\cos(\theta-208^\circ)$	$+0\cdot7\cos 2(\theta-20^\circ)$	$+\cdot29\cos 3(\theta-64^\circ)$	$+\cdot12\cos 4(\theta-94^\circ)$
$\frac{B-\text{Neb.}}{\text{Maximum}}$	·82	·09	·40	·09

"The first harmonics are discordant, but the others agree well. Not only are the phases similar, but the order of magnitude of the coefficients is the same, viz. III., IV., II. Both agree in having a very weak second harmonic. The arithmetical progression is again in evidence.

Algiers +1°.

(Reference, *M.N.*, June 1912, p. 700.)

	I.	II.	III.	IV.
	°	°	°	°
Algiers plates	45	77	131	208
Neb. zones 55° – 90°	208	20	184	184
Neb. zone 90° – 125°	220	84	131	192
Weighted mean	217	75	157	190
$\frac{\text{A} - \text{Neb.}}{\text{Maximum}}$	·96	·02	·43	·40

"The first harmonic is quite discordant; the fourth and third are shaky. The order of importance of the coefficients agree, viz. IV., III., II.

"The whole thing looks interesting, but I am unable to carry it any further as I am leaving for the Front this week, so I hope you will not mind this very scrappy note. I shall look forward to reading, in due course, I hope, your promised discussion of the variable ratio of faint stars to bright when more zones are available. Yours truly, F. G. Brown."

No. 8 Platoon, O Company,
5th City of London Rifles,
Haywards Heath.
1915 *March* 1.

On the Errors of Measurements on Astro-Photographic Plates.

By Winifred Gibson.

(*Communicated by the Secretaries.*)

In a paper published in the *Monthly Notices of the Royal Astronomical Society* for December 1911, I gave the results of three series of measures of star images on photographic plates. The main object was to discover whether any particular grouping of two orientations gave the best compensation for the personal error, or whether readings in four orientations were always advisable where great accuracy is required.

In the present paper I have shown:

(i.) That cases do occur in which the personal error is best eliminated by taking readings in adjacent orientations rather than in opposite ones.

(ii.) That different observers measuring the same star image may find different groupings of orientations give the best results.

(iii.) That the same observer may find different groupings of orientations the best when measuring different star images.

For all these reasons it seems advisable to take readings in all four orientations of the plate if great accuracy is required in the final measure.

I first obtained readings from three different observers A, B, and C on ten star images on the same plate. The measures were those of the X co-ordinates of these images referred to the left-hand réseau line as axis. The unit of measurement was the length of a réseau line, *i.e.* $\frac{1}{2}$ cm. throughout.

Each star image was measured seven times by each observer in four orientations of the plate. In each measure the image was twice bisected, so that the mean finally obtained from readings in all four orientations is the result of fifty-six bisections.

The means of all the twenty-eight readings of each observer on each image are given in Table I., and also the differences of these means from the means of the seven readings in each orientation.

For the sake of conformity I have used my former notation $e_{o,\,0}$, $e_{o,\,90}$, $e_{o,\,180}$, $e_{o,\,270}$ for these residuals, but it must be remembered that since in this case they are based on seven measures only, they in all probability include a considerable amount of the accidental error $e_{a,\,0}$, $e_{a,\,90}$, etc., and cannot therefore be regarded as wholly systematic.

The part of the error referred to in my first paper as $e'_{a,\,0}$, $e'_{a,\,90}$, etc., is of course simply the variable part of the personal error, the variation of which is more likely to be apparent in a long series of observations than in a short one. On the other hand, the *purely* accidental error $e''_{a,\,0}$, etc., can only be eliminated if a considerable number of readings are taken.

As a rough test, however, of the best pair of orientations for each observer, I calculated the values of

$$\sum\left(\frac{e_{o,\,0}+e_{o,\,90}}{2}\right)^2, \quad \sum\left(\frac{e_{o,\,0}+e_{o,\,180}}{2}\right)^2, \text{ and } \sum\left(\frac{e_{o,\,90}+e_{o,\,180}}{2}\right)^2$$

for the ten star images. The results are given below:—

	A.	B.	C.
$\sum\left(\frac{e_{o,\,0}+e_{o,\,90}}{2}\right)^2$	16·5	245·2	369·7
$\sum\left(\frac{e_{o,\,0}+e_{o,\,180}}{2}\right)^2$	158·5	482·7	207·7
$\sum\left(\frac{e_{o,\,90}+e_{o,\,180}}{2}\right)^2$	591·5	281·7	1092

TABLE I.

Readings of three Observers, A, B, and C, on ten Star Images on the same Plate. (*Star images of the most varied character possible were selected.*)

	A.	B.	C.
1. Mean reading	·9052	·9070	·9079
$e_{o,\,0}$	+ ·0018	+ ·0018	+ ·0013
$e_{o,\,90}$	− 17	− 14	− 3
$e_{o,\,180}$	+ 3	− 6	+ 2
$e_{o,\,270}$	− 4	+ 2	− 11

TABLE I.—*continued.*

	A.	B.	C.
2. Mean reading	·2857	·2873	·2886
$e_{0,0}$	− ·0008	− ·0005	− ·0024
$e_{0,90}$	+ 11	− 10	+ 3
$e_{0,180}$	+ 1	+ 10	+ 9
$e_{0,270}$	− 4	+ 5	+ 12
3. Mean reading	·6715	·6718	·6690
$e_{0,0}$	− ·0005	− ·0012	+ ·0001
$e_{0,90}$	+ 6	+ 5	− 9
$e_{0,180}$	+ 8	− 16	− 10
$e_{0,270}$	− 9	+ 23	+ 18
4. Mean reading	·8632	·8639	·8591
$e_{0,0}$	− ·0012	+ ·0007	+ ·0014
$e_{0,90}$	+ 12	+ 5	− 19
$e_{0,180}$	+ 15	+ 4	− 21
$e_{0,270}$	− 15	− 16	+ 26
5. Mean reading	·2736	·2739	·2690
$e_{0,0}$	− ·0004	− ·0007	+ ·0024
$e_{0,90}$	+ 6	− 7	− 8
$e_{0,180}$	+ 8	+ 0	− 22
$e_{0,270}$	− 10	+ 14	+ 6
6. Mean reading	·5389	·5381	·5391
$e_{0,0}$	− ·0010	− ·0003	− ·0012
$e_{0,90}$	+ 16	+ 2	+ 14
$e_{0,180}$	+ 8	+ 2	+ 4
$e_{0,270}$	− 14	− 1	− 6
7. Mean reading	·3763	·3767	·3792
$e_{0,0}$	− ·0005	− ·0002	− ·0018
$e_{0,90}$	+ 4	+ 7	+ 14
$e_{0,180}$	+ 7	− 5	+ 11
$e_{0,270}$	− 6	+ 0	− 7
8. Mean reading	·1359	·1387	·1380
$e_{0,0}$	− ·0009	+ ·0004	− ·0014
$e_{0,90}$	+ 7	+ 13	− 8
$e_{0,180}$	− 1	− 15	+ 24
$e_{0,270}$	+ 3	− 2	− 2

TABLE I.—*continued.*

	A.	B.	C.
9. Mean reading	·0500	·0519	·0493
$e_{o,\,0}$	+ ·0002	− ·0018	+ ·0004
$e_{o,\,90}$	− 3	+ 14	+ 1
$e_{o,\,180}$	− 1	+ 4	− 7
$e_{o,\,270}$	+ 2	− 0	+ 2
10. Mean reading	·0955	·0958	·0934
$e_{o,\,0}$	− ·0007	− ·0016	− ·0001
$e_{o,\,90}$	+ 10	+ 19	− 7
$e_{o,\,180}$	+ 6	− 5	− 4
$e_{o,\,270}$	− 9	+ 2	+ 12

These measures were taken through the courtesy of Professor Turner by means of the plate-measurer in the Oxford University Observatory.

It is obvious at once that in the case of A very much better compensation is obtained by turning the plate through a right angle than by turning it through 180° and taking readings in opposite orientations. It is not the case, however, that *any* pair of adjacent orientations give better compensation than opposite ones. A combination of readings in 0° and 90° gives markedly better results than any other grouping, but opposite orientations appear distinctly preferable to any other pair of adjacent ones.

In B's case, again, the two orientations 0° and 90° give results most free from errors, but for this observer *any* pair of adjacent orientations would seem to be preferable to opposite ones.

For C, on the other hand, a combination of readings in opposite orientations is distinctly preferable.

Since the same star images were measured by all three observers, it seems that the particular pair of orientations giving the best results is a function rather of the personality of the observer than of the star image itself.

It is noticeable that in seven out of ten cases A obtains for the residuals in the four orientations a sign sequence − + + −, giving some compensation when readings are taken in opposite orientations as well as when the plate is turned through a right angle. In only two cases, however, do opposite orientations give the best results for this observer.

The results for A in particular point to the fact that there may be a very high correlation between the *personal* errors of any observer in two orientations. A high positive correlation between the personal errors in two orientations would show that there was no compensation to be obtained by taking the mean of readings in these particular orientations.

A high negative correlation, on the other hand, would show that good compensation was, in general, to be expected.

The pair of orientations giving the highest negative correlation coefficient for the personal errors would be the best for that particular observer, but a satisfactory measure of the correlation could only be obtained by taking measures (in four orientations) on many star images, each complete measure being repeated a considerable number of times in order to eliminate, as far as possible, the accidental error from the mean.

In this paper I have not attempted to show anything more than the fact that cases certainly do arise in which turning the plate through a right angle gives better compensation as a rule than turning through 180°. But even when two orientations are very decidedly the best for a particular observer exceptions are likely to occur, and it is not safe therefore to assume that the result will be trustworthy unless four orientations are taken.

Table I. shows clearly that different observers, when measuring the same star image, may find different groupings of orientations give the best results, and also that the same observer may find different groupings the best for different star images, but I am inclined to think that an experienced observer using the same plate-measurer for a considerable time may find that the same two orientations give the best results in the majority of cases.

The two best orientations are not necessarily opposite ones, however, and if measures of extreme accuracy are required, readings in all four orientations are advisable.

Errors due to the Film.—I have made some attempt to find what effect the irregularities of the film itself have upon the errors of measurement. I used one of the Cambridge plates, on which there were four images of each star photographed, for this purpose, and by measuring each image of a particular star in turn, I hoped to obtain some idea as to the amount of error due to this cause.

The particular image chosen was slightly irregular in shape, but well defined.

I measured the first image fifty times, in order to obtain some idea as to the number of readings necessary for the elimination of the purely accidental error, and also the amount of variation to be expected in the personal error itself during a fairly long series of observations.

The measures were taken under fairly uniform conditions of time and lighting.

The second and third images were only measured twenty times. The fourth image was on the réseau line, and I did not therefore attempt to measure it.

The different orientation readings were taken in the order 0°, 180°, 90°, 270° for the first two images, in order to avoid rotating the scale and eyepiece more than once in each complete observation. For the third image, readings were taken in the order 0°, 90°, 180°, 270°, and the eyepiece was therefore turned through a right angle after each measure in any one orientation.

In the tables given below I have shown the results of these

measures. In Table II. the mean readings in each orientation are given for the different star images, and the results of taking 10, 20, . . . readings are compared.

In Table III. the first column gives the mean reading of the measures in all four orientations. In the remaining columns the residuals formed by subtracting the readings in Table II. from those in the first column of Table III. are given.

TABLE II.

		0.	90.	180.	270.
10 readings	1st image	·5896	·5870	·5869	·5893
20 ,,		·5897	·5871	·5865	·5895
30 ,,		·5894	·5870	·5864	·5896
40 ,,		·5895	·5870	·5863	·5894
50 ,,		·5897	·5869	·5862	·5890
10 readings	2nd image	·3940	·3909	·3907	·3940
20 ,,		·3939	·3913	·3905	·3938
10 readings	3rd image	·1884	·1847	·1849	·1872
20 ,,		·1877	·1849	·1848	·1874

TABLE III.

		Mean of Whole.	$e_{0,\,0}$.	$e_{0,\,90}$.	$e_{0,\,180}$.	$e_{0,\,270}$.
10 readings	1st image	·5882	+ ·0014	− ·0012	− ·0013	+ ·0011
20 ,,		·5882	+ 15	− 11	− 17	+ 13
30 ,,		·5881	+ 13	− 11	− 17	+ 15
40 ,,		·5880	+ 15	− 10	− 17	+ 14
50 ,,		·5879	+ 18	− 10	− 17	+ 11
10 readings	2nd image	·3924	+ ·0016	− ·0015	− ·0017	+ ·0016
20 ,,		·3924	+ 15	− 11	− 18	+ 14
10 readings	3rd image	·1863	+ ·0021	− ·0016	− ·0014	+ ·0009
20 ,,		·1862	+ 15	− 13	− 14	+ 12

Table II. shows clearly the very marked effect of the orientation on the personal error, and also the fact that the variation of this personal error in any one orientation is not very great, though the series of observations extended over a considerable time.

Provided that readings are taken in all four orientations, ten complete observations seem to be sufficient for the elimination of the purely accidental error from the mean. Since the probable error of this result, however, is $\frac{1}{\sqrt{5}}$ times that of the error of personality e in any one orientation, I thought it advisable to take at least twenty readings in order to obtain trustworthy results for these latter quantities.

If we compare the values of the $e_0 s$ obtained from twenty readings in the four orientations for the different images, it is evident that for the first and second the agreement is excellent, and the fact that these images are on different parts of the plate has apparently no effect on the personal error made in measuring them.

For the third image the agreement is not quite so good, especially in orientations 90° and 180°, but the differences still fall within the limits of the fluctuations of the personal error for the first image.

It is possible that with a less good plate more marked differences would be shown in the results for the different images, but, as far as they go, the readings given in Tables II. and III. suggest that the effect of the irregularities of the film itself on the image, and consequently on the error of personality which occurs when it is measured, may be inconsiderable.

The Rotation of the Earth. By H. Glauert, B.A., Isaac Newton Student, Cambridge University.

In his address to the British Association in Australia last year, Professor E. W. Brown * exhibited diagrams of the deviations in longitude of the Moon from its theoretical orbit, and placed at their side similar curves for Mercury and the Sun. These curves possessed a certain similarity, and Professor Sir Joseph Larmor † has recently published an investigation of the effect of surface disturbances on the rate of rotation of the Earth and the corresponding orbital acceleration of the Moon. A change in the rate of rotation of the Earth causes a corresponding change in the unit of time, which becomes apparent as an error of position of any body moving fast enough to show the effect. Recent years are especially favourable for the detection of any such effect, as the Moon's error is large and steadily increasing. In the present paper the errors in longitude of Mercury, Venus, and the Sun are investigated for indications of minute changes in the rate of rotation of the Earth.

An observed error e in geocentric longitude of a planet leads to the equation—

$$e = A\delta L + B\delta l,$$

where δL and δl are respectively the errors in heliocentric longitude of the planet and the Earth. The values of e, A, and B were taken from the Greenwich Observations for various dates in each year, and the equations were weighted according to the number of

* *Brit. Assoc. Report*, Australia, Sept. 1914: "Address on Cosmical Physics."

† *R.A.S., Monthly Notices*, vol. lxxv. p. 211.

observations (n) in right ascension. Assuming the errors to be due to changes in the rate of rotation of the Earth, leading to non-uniformity in the measurement of time, $\delta l = p\delta L$, where p is the ratio of the mean motions of the Earth and planet. For Venus p has the value 0·615, and for Mercury 0·241. In this way the fundamental equation becomes

$$e = (A + pB)\delta L = a\delta L,$$

while for the mean error of a year,

$$\delta L = \frac{\Sigma nae}{\Sigma na^2} \quad \text{and} \quad \delta l = p\delta L.$$

The period considered has been from 1865 to 1914, and for the greater part of this period two corrections had to be applied. The former represents the changes in the Greenwich clock star places, and the latter the difference between the tables of Le Verrier and Newcomb, which were successively used in calculating the tabular positions of the planets in the *Nautical Almanac*.

The corrections to the Greenwich clock star places before the year 1903 were drawn from a paper of Professor E. W. Brown.* The mean correction c in longitude is derived from that in right ascension by multiplying by the cosine of the obliquity. As the error e is the excess of the tabular over the observed value, c must be subtracted from e.

Period.	$\Delta\alpha$.	c.
	s	″
1862–69	+0·024	+0·33
1870–77	0·034	0·47
1878–88	0·029	0·40
1889–92	0·037	0·51
1893–98	0·049	0·67
1899–1902	0·054	0·74

Before 1901 Le Verrier's tables were used in calculating the tabular places of Venus, Mercury, and the Sun. The differences of mean longitude (Le Verrier – Newcomb) for the three bodies are—

Sun . . .	+0″·71	+0″·82T	+0″·018T²
Venus . .	+1·74	+3·29T	+0·0141T²
Mercury . .	+0·09	+0·82T	+0·045T²

where T is measured in centuries from 1900·0. The results for Venus and the Sun were taken from papers of Dr. Cowell †; that for Mercury was derived directly from the formulæ given by Le Verrier and Newcomb for the mean longitude.

* *R.A.S., Monthly Notices*, vol. lxxiii. p. 700.
† *Ibid.*, vol. lxvi. pp. 304, 319.

Writing D for this difference in the case of the planet and d in that of the Sun, δL and δl must be replaced respectively by $D + \delta L$ and $d + \delta l$. When both corrections are applied, the fundamental equation becomes

$$E = a\delta L,$$

where

$$E = e - (DA + dB + c).$$

In the case of the Sun, the mean error in longitude for each month of the year is given in the Greenwich Observations. The mean of these for each year was taken and corrected by subtracting the quantity $(d + c)$.

In this way three determinations of δl for each year were obtained, but to smooth out accidental errors the weighted mean of each successive group of three years was taken in each set of results. The mean of these determinations, giving half weight to the results derived from Mercury, was taken as the resulting error in longitude of the Earth. The four sets of values are tabulated below and are also represented in the diagrams on page 493. The corresponding curve for the Moon is also given, the values being due to Professor Brown.*

Values of δl.

Year.	Sun.	Venus.	Mercury.	Mean.
	″	″	″	″
1866	−0·81	−0·43	−0·47	−0·59
1867	−0·88	−0·18	−0·17	−0·46
1868	−0·80	−0·20	+0·13	−0·37
1869	−0·73	−0·04	+0·25	−0·26
1870	−0·73	+0·11	+0·25	−0·20
1871	−0·83	+0·09	+0·05	−0·29
1872	−0·75	+0·03	−0·03	−0·29
1873	−0·57	+0·11	−0·19	−0·22
1874	−0·31	+0·08	+0·13	−0·07
1875	−0·26	+0·39	+0·15	+0·08
1876	−0·38	+0·21	+0·23	−0·02
1877	−0·36	+0·15	+0·05	−0·07
1878	−0·33	−0·17	+0·04	−0·19
1879	−0·18	−0·16	−0·03	−0·14
1880	−0·20	+0·04	−0·20	−0·10
1881	−0·25	+0·06	−0·61	−0·20
1882	−0·36	−0·04	−0·21	−0·20
1883	−0·37	−0·05	−0·15	−0·20
1884	−0·31	−0·10	+0·08	−0·15

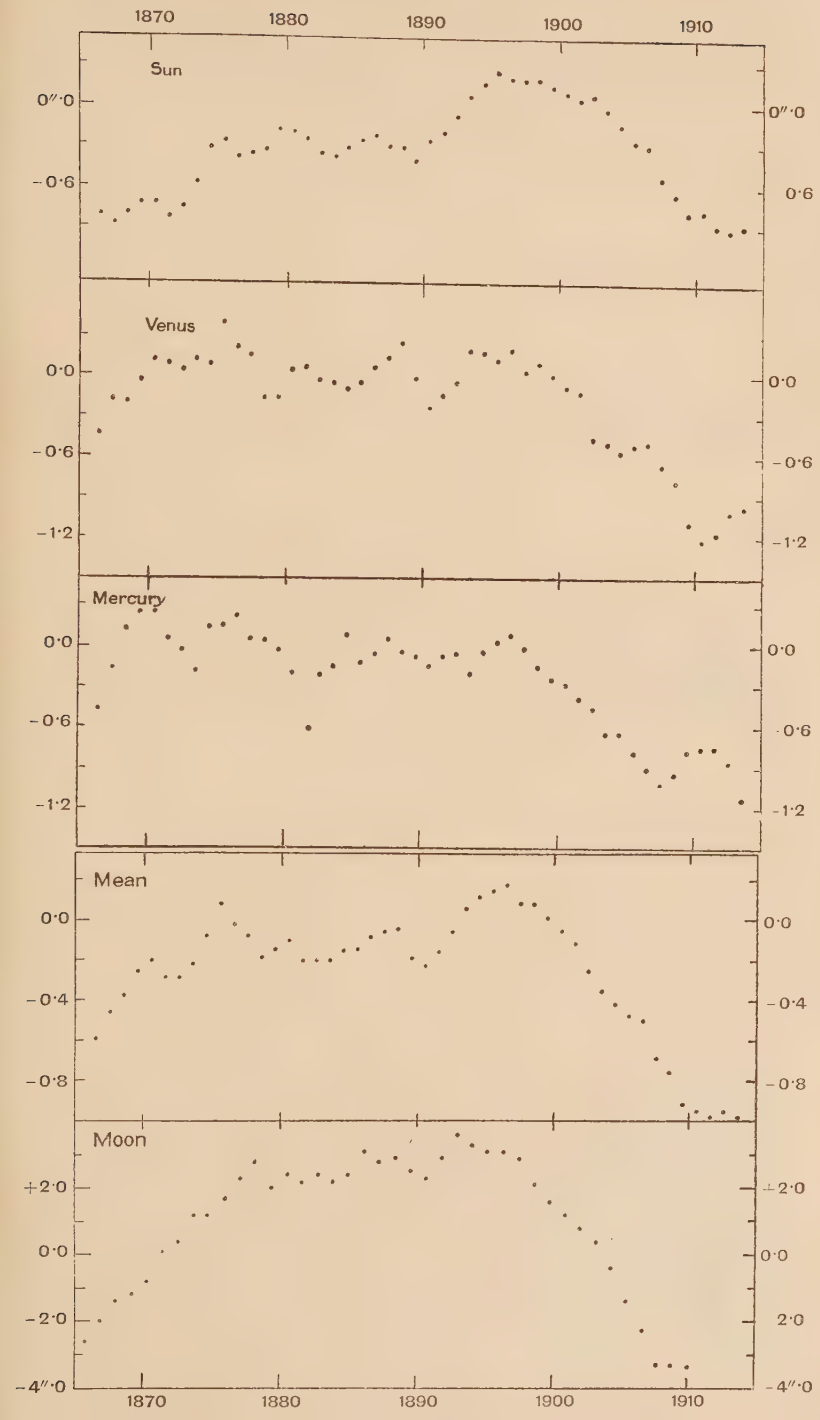
* *R.A.S., Monthly Notices*, vol. lxxiii. p. 692.

Values of δl—continued.

Year.	Sun.	Venus.	Mercury.	Mean.
1885	−0″·25	−0″·05	−0″·12	−0″·14
1886	−0·22	+0·06	−0·06	−0·08
1887	−0·30	+0·13	+0·05	−0·06
1888	−0·31	+0·24	−0·05	−0·04
1889	−0·41	−0·02	−0·07	−0·19
1890	−0·26	−0·24	−0·15	−0·23
1891	−0·20	−0·15	−0·08	−0·16
1892	−0·07	−0·05	−0·06	−0·06
1893	+0·07	+0·18	−0·21	+0·06
1894	+0·16	+0·17	−0·05	+0·12
1895	+0·25	+0·12	+0·03	+0·15
1896	+0·20	+0·19	+0·08	+0·17
1897	+0·19	+0·02	−0·02	+0·08
1898	+0·20	+0·09	−0·16	+0·08
1899	+0·14	0·00	−0·25	+0·01
1900	+0·09	−0·08	−0·28	−0·05
1901	+0·04	−0·12	−0·39	−0·11
1902	+0·08	−0·47	−0·47	−0·25
1903	−0·03	−0·51	−0·65	−0·35
1904	−0·14	−0·57	−0·65	−0·41
1905	−0·27	−0·52	−0·79	−0·47
1906	−0·30	−0·50	−0·91	−0·50
1907	−0·53	−0·67	−1·03	−0·69
1908	−0·65	−0·78	−0·95	−0·76
1909	−0·79	−1·10	−0·79	−0·91
1910	−0·78	−1·22	−0·76	−0·95
1911	−0·89	−1·17	−0·76	−0·98
1912	−0·91	−1·02	−0·87	−0·95
1913	−0·89	−0·98	−1·14	−0·98

The final curve for the errors of the Earth shows a very considerable similarity to that of the Moon, and it is important to note in this connection that if the effect is due to changes in the rate of rotation of the Earth, it should be revealed by correlation in the rate of change of the errors, not necessarily by the errors themselves, since the latter depend on the assumed mean motion and longitude at epoch.

If δl and $\delta l'$ be the errors of longitude of the Moon and Earth,


1870
1880
1890
1900
1910
Sun
0″·0
−0·6
0″·0
0·6
Venus
0·0
−0·6
−1·2
0·0
−0·6
−1·2
Mercury
0·0
−0·6
−1·2
0·0
·0·6
−1·2
Mean
0·0
−0·4
−0·8
0·0
−0·4
−0·8
Moon
+2·0
0·0
−2·0
−4″·0
+2·0
0·0
2·0
−4″·0
1870
1880
1890
1900
1910

n and n' their mean motions, and N the angular velocity of rotation of the Earth, then due to changes in n and N—

$$\frac{d}{dt}(\delta l) = n\left(\frac{\delta \mathrm{N}}{\mathrm{N}} - \frac{\delta n}{n}\right)$$
$$\frac{d}{dt}(\delta l') = n'\frac{\delta \mathrm{N}}{\mathrm{N}}.$$

Thus, if n does not vary—

$$\frac{d}{dt}(\delta l) = (13{\cdot}4)\frac{d}{dt}(\delta l');$$

but if the change of momentum of the Earth is compensated for by changed orbital velocity of the Moon,*

$$\frac{\delta n}{n} = \frac{23}{36}\frac{\delta \mathrm{N}}{\mathrm{N}},$$

and so

$$\frac{d}{dt}(\delta l) = (4{\cdot}8)\frac{d}{dt}(dl').$$

Now for the two periods 1866–1875 and 1898–1909, the rates of change of δl and $\delta l'$ are fairly steady, and approximations to the curves in the form $y = mx + c$ may be obtained. The values of m and m' are derived:

Moon . .	$m_1 = {\cdot}427,$	$m_2 = -{\cdot}595;$
Earth . .	$m_1' = {\cdot}0565,$	$m_2' = -{\cdot}0873.$

It is to be noted that $\frac{m_2}{m_1} = -1{\cdot}4$ and $\frac{m_2'}{m_1'} = -1{\cdot}5$, showing the close similarity of the curves. Also $\frac{m_1}{m_1'} = 7{\cdot}6$ and $\frac{m_2}{m_2'} = 6{\cdot}8$, indicating that for the mean of the periods considered—

$$\frac{d}{dt}(dl) = (7{\cdot}2)\frac{d}{dt}(\delta l'),$$

and this factor 7·2 lies between the two theoretical values 13·4 and 4·8.

Mr. Thackeray has provided data of the errors in right ascension of the Sun for different observers for the period 1894–1914, which give a useful check on the effects of personality. The values of m_2' for the nine principal observers are given below, all of whom, except possibly E., give clear indications of the general slope of the curve.

* Professor Sir Joseph Larmor, *R.A.S., Monthly Notices,* vol. lxxv. p. 218.

Observer.	Period.	m_2'.
H.	1894–1901	– ·09
A. C.	1894–1905	– ·19
B.	1894–1905	– ·05
R.	1896–1902	– ·21
W. B.	1896–1903	– ·15
W.	1902–1914	– ·08
J. S.	1902–1908	– ·25
R. C.	1903–1914	– ·19
E.	1904–1912	– ·02

The value of m_2', as deduced from the results of all observers, is $-\cdot 087$, while the mean of the above values, weighted according to the length of the period, is $-\cdot 131$. This latter value leads to $\frac{m_2}{m_2'} = 4\cdot 5$, which is very close to the theoretical value 4·8.

While the results for separate observers show a certain divergence, the general effect is clearly indicated, and it appears that the effect of a large number of observers is to smooth out, rather than to accentuate the errors. From the results obtained, it appears that the errors in longitude of the Moon and the three bodies considered in this paper may be accounted for by a rather irregular variation in the rate of rotation of the Earth, the change of momentum being partially or entirely compensated for by a corresponding change in the mean motion of the Moon.

It is of interest to note the magnitude of the changes in the length of the day indicated by the above results. The quantity m' is the value of $\frac{d}{dt}(\delta l')$ expressed in seconds of arc per year, and so if x seconds of time be the excess of the length of the day at any time over its standard value, $15x = -m'$.

1866–1875	$m_1' = +\cdot 0565$	$x = -0^s\cdot 0038$
1898–1909	$m_2' = -\cdot 0873$	$x = +0^s\cdot 0058$

Thus an increase in the length of the day of $0^s\cdot 01$ is indicated in a third of a century. Also for the latter period the value of m_2' leads to $\frac{\delta N}{N} = -\frac{6\cdot 7}{10^8}$, which happens to agree with the value mentioned by Professor Sir Joseph Larmor * in a recent paper.

In conclusion, I wish to express my indebtedness to Professor Eddington for suggesting the research and also for his continual interest and advice during the course of the work.

* *R.A.S. Monthly Notices*, vol. lxxv. p. 218.

The Sun-spot and the Solar Corona of 1914 *August* 21.

By the Rev. A. L. Cortie, S.J.

1. In former papers (*Monthly Notices, R.A.S.*, lxxiii. pp. 431–436, pp. 539–543) evidence has been furnished to show that regions of long-continued sun-spot activity were associated, in four cases, with bundles of divergent streamers in the solar corona. The following table gives a summary of the results so far attained, to which are added those connected with the present investigation.

Corona.	Duration.	*Disturbed Area of Sun-spots.* Mean Longitude.	Mean Latitude.
		° °	° °
1893 Apr. 16	1893 Jan. 16–Nov. 16	49·3 to 14·9	− 7·5 to −13·1
1898 Jan. 22	1898 Jan. 19–July 29	132·7 ,, 117·8	− 8·8 ,, −13·7
1905 Aug. 30	1905 May 11–July 7	72·0 ,, 52·3	−12·4 ,, − 18·5
1908 Jan. 3	1907 May 5–1908 Jan. 8	258·7 ,, 240·3	+ 4·9 ,, + 9·1
1914 Aug. 21	1914 Mar. 30–Nov. 14	71·4 ,, 56·3	+27·7 ,, +17·2

In the four cases so far discussed the regions of solar activity happened to be situated near to the Sun's limb on the dates of total eclipse. It is not claimed that the treatment was exhaustive, as there may well have been other streamers connected with regions of spot-activity which were not placed in these more favourable positions for investigation. In particular, in the case of the eclipse of 1893 April 16, Professor Schaeberle has discussed (*Contributions from the Lick Observatory*, No. 4) several other regions of activity, situated on the Sun's disc, which were presumably connected with streamers in the Sun's corona.

2. *The Sun-spot.*—The sun-spot which crossed the Sun's disc August 13–26 was the largest spot which had appeared in the new cycle of spot-activity. Its position on August 21 was longitude 67° 18′, north latitude 18° 18′. We must go back to 1910 October to find any spot approaching it in size. When it appeared on the limb on August 13, it was a regular spot of Type IV., the preliminary stages of its life-history having been passed on the invisible hemisphere of the Sun. It attained a disc-area of 6·5 units ($\frac{1}{3000}$ of the visible hemisphere), and was the largest spot of the year. Through the kindness of Professor Hale, Professor Frost, and Professor Newall, I have been supplied with spectro-heliograms of the spot and the accompanying flocculi in hydrogen and calcium radiations, from the Mount Wilson, Yerkes, and and Kodaikánal Observatories. And besides the Stonyhurst series of drawings, through the kind offices of the Astronomer Royal I have at my disposal a fine series of photographs of the spot and the surrounding faculæ obtained at Greenwich. The area of faculæ and flocculi surrounding the spot is of great extent. The hydrogen flocculi cover a region from longitude 69·9 to 54·7, and

from latitude +27·1 to +16·3. In this area of hydrogen flocculi there is, at longitude 59·3 and latitude +21·9, on the Mount Wilson spectroheliogram of August 21, and less markedly in that of August 22, a dark patch which evidently marks the position of the following spot of the group, when it passed through the two-spot stage, or Type II. of its life-history on the invisible hemisphere of the Sun. It will be interesting to ascertain whether this portion of the mass of flocculi exhibited an opposite polarity to that of the spot itself in the magnetic field.

The bright calcium flocculi, as shown on the spectroheliograms from Kodaikánal and Yerkes, are of much greater extent than the hydrogen flocculi. The limits of longitude are approximately 70°·0 to 52°·0, and of latitude from +28°·5 to +15°·4. The dark patch in the hydrogen flocculi, already alluded to, is well shown both in the Yerkes and Kodaikánal spectroheliograms, in calcium light, of August 22.

The sun-spot of 1914 August 21 was not an isolated disturbance, but one of a series of disturbances in the same locality on the Sun. Not only was it the largest spot of the new cycle and of the year, but it occurred in that region of the Sun which was first disturbed at the advent of the new cycle, and was most disturbed during the year. Confining ourselves to the limits of longitude and latitude of the flocculi accompanying the spot of August 21, we find that the same region was disturbed from March 30 to November 15, intermittently, by five spot-groups, four of which were among the most important disturbances of the year. The date March 30 also, and the group that then appeared on the E limb, marks the beginning of the recrudescence of spot-activity, for the first three months of 1914 were almost as barren of sun-spots as the year of absolute minimum 1913. The following table gives the mean positions of these spot-groups from the Stonyhurst drawings :—

No.	Date.	Mean Heliographic.	
		Longitude.	Latitude.
	1914.	°	°
1	Mar. 30–Apr. 11	71·4	27·7
2	Apr. 27–May 2	56·3	26·1
3	Aug. 13–Aug. 26	66·5	19·0
4	Sept. 10–Sept. 22	65·3	17·2
5	Nov. 7–Nov. 15	69·3	23·3

The groups 1, 3, 4, 5 were the more important groups.

3. If any connection existed between the streamers of the corona of 1914 August 21 and a disturbed spot-region, all the evidence of the life-histories of the spots pointed to the particular region occupied by the large spot of the same date. It is also worthy of note that the total solar eclipse of 1914 is one of the very few that have been observed when the spot-activity was beginning to increase. The mean daily area of spots for 1914 from the Stonyhurst drawings was 0·82. That for 1913 was only 0·04,

so that the increase, which took place mainly in nine months, was slightly more than twentyfold.

4. *The Corona.*—The form of the corona of 1914 was that of the minimum type in its main features, especially as seen by the naked eye. But it was differentiated from a purely minimum type by some bright streamers which were superposed upon the equatorial wings. It has already been described in the Preliminary Report of the Total Solar Eclipse (*Monthly Notices, R.A.S.*, lxxv. p. 116). The features we wish particularly to call attention to in the present investigation are the following:—In the N.E. quadrant there was a remarkably bright sheaf of rays, with two brighter ribs at P.A. 48°·0 and 52°·3 which extended as far as 26′ from the limb. In the N.W. quadrant was a curved streamer, with two prominent tips or horns at P.A. 332°·9 and 335°·7. Above this streamer at P.A. 357°·4 was a very bright single curved ray, with the curvature towards the equator, which reached to about 7′ from the Sun's limb. These features mainly contributed to differentiate the corona from that of a minimum type, and gave it the appearance of an inchoate intermediate type. As already remarked, in the coronas of 1893, 1898, 1905, 1908, the regions of selected spot-activity were close to the Sun's limb on the days of eclipse. Hence it was only necessary to show that the "radiant" area, or "apex" of the converging streamers, was situated in such a region of spot-activity, to establish a probable connection between the phenomena. The weight of the evidence for a real, and not merely a fortuitous, connection is increased by its occurring in four cases, and not merely in one case.

In the present instance the heliographic longitude of the spot being 67° 18′, it was situated some 68° from the Sun's limb. A method of solution of the problem as to whether the streamers detailed above were connected with the spot was suggested by Mr. F. Henroteau, a Belgian refugee, now resident at Stonyhurst, and formerly of the Royal Observatory, Uccle. He has also made all the necessary measurements and constructions connected with the work. According to Pascal's theorem, if the intersections of opposite pairs of sides of a hexagon are collinear, the six points lie on a conic section. The method adopted was, therefore, to plot on squared paper five points, measured on a ray which it was suspected was connected with the spot, and to take the centre of the spot as a sixth point. With these six points a hexagon was constructed, and the test of collinearity of the intersection of the directions of its opposite pairs of sides, showed whether the five points on the ray, and the sun-spot were situated on a conic. Six rays were so treated, those detailed above and one also in the S.W. quadrant. All except the last mentioned stood the test, and as five projected conic sections meet in the region of the spot, though not in the actual spot itself, it is considered that the evidence is sufficient to establish a connection between these streamers and the spot. The measures were made on squared paper superposed on enlarged photographs of the eclipsed

Sun, on a scale of six inches to the Moon's diameter. The relative positions of the five points measured on a ray and the sun-spot were set down on a still larger scale on squared mm. paper, before constructing the hexagons. A curve of errors was also drawn for each case to ascertain how far the point O, representing the spot, could be shifted without altering the collinearity of the three points of intersection of the opposite pairs of sides of the hexagon.

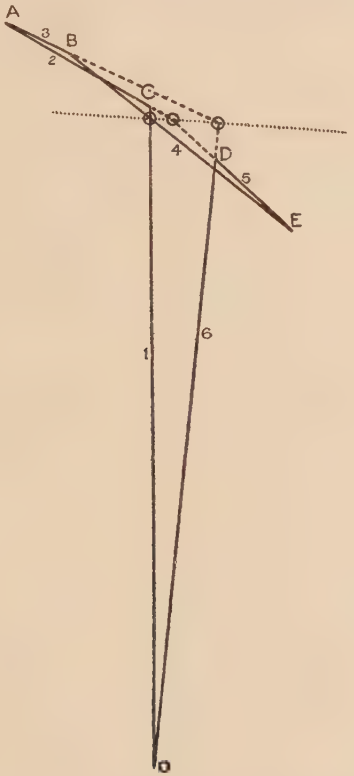


FIG. 1.—A, B, C, D, E are five points on a bright ray at position-angle 357°·4. O is the centre of the spot. The small circles, which are collinear, are the intersections of the opposite sides of the hexagon.

In all the cases except one this point could be shifted, but the curve of errors showed whether it could be shifted beyond the region of disturbance, such as that of the flocculi, for instance, connected with the spot.

In the case of the bright single ray at P.A. 357°·4 the solution is rigid, as the collinearity of the points of intersection is disturbed by altering the position O of the spot. This case is illustrated in fig. 1 above. In the case of the two tips of the streamer in the N.W. quadrant, the solution is not so rigid, as the position of

the centre of the spot, the point O, can be altered within limits without disturbing the collinearity of the points of intersection of opposite sides of the hexagon. This is due to the fact that the angles of the hexagon are very acute or very obtuse. But the limits through which O can be shifted lie within the region of the flocculi, so that it is true to say that the streamer emanates from the region of the spot, though not necessarily from the spot itself.

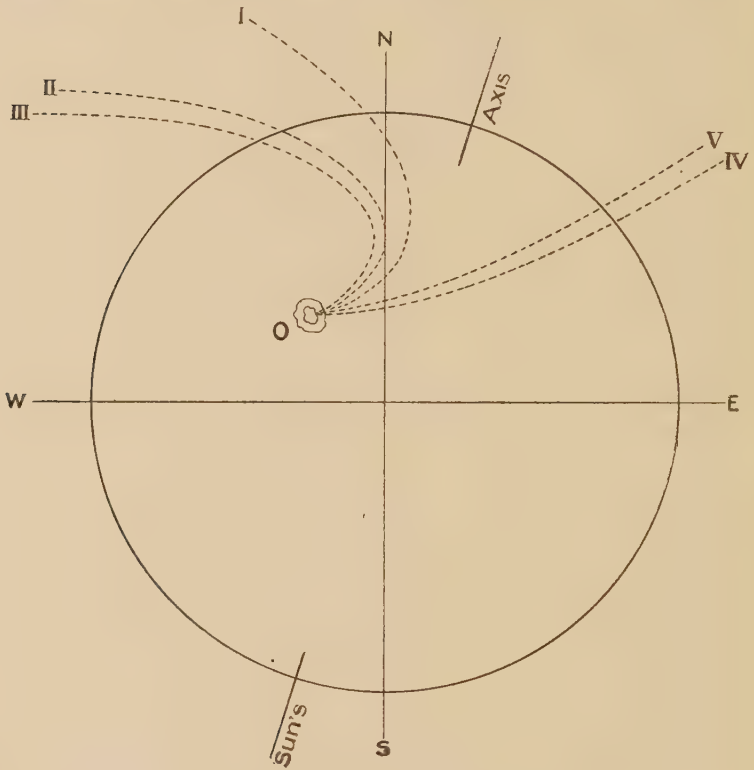


Fig. 2.—Curves I., II., III. on the West; Curves IV, V on the East; O is the spot.

The very bright streamer in the N.E. quadrant points almost directly at the spot. It is only necessary to give the directions of the bright ribs a slight curvature to lead them directly through the spot. But this renders the solution still less rigid than in the preceding cases. It is possible to shift the position of the centre of the spot considerably without altering the collinearity of the three points of intersection of opposite sides of the hexagon. But even in this case the directions of the rays pass through the region of disturbances connected with the spot.

We may for convenience (fig. 2) number the curves in the

order of decreasing value of P.A. Applying the geometrical process of reciprocation, and effecting a graphical solution, curve I. is a very elongated eclipse, curves II. and III. are hyperbolæ, and curves IV. and V., being nearly straight lines, are the projections in all probability of conics seen almost edgewise. Since these five curves intersect one another in the region of the spot, it is highly probable that the disturbed region of the Sun's surface, which included the sun-spot of 1914 August 21, was the seat of the emanation of widely divergent streamers, represented in projection by the five conics discussed above.

This result, combined with those established for the eclipses of 1893, 1898, 1905, and 1908, lends additional weight to the opinion that the mode of propagation of matter from a centre of disturbance on the Sun, which may be supposed to be operative in the production of terrestrial magnetic storms, is in the form of divergent streamers, and not in that of a single discharge, on the analogy of a stream of cathode rays.

5. The following table gives a summary of a comparison of the spotted state of the Sun, and the terrestrial magnetic phenomena for the years 1912–14, from the Stonyhurst records:—

	Mean Daily.		Number of Magnetic Disturbances.	
Year.	Spot-Area.	Declination Range.	Moderate.	Greater.
		′		
1912	0·22	8·1	18	1
1913	0·04	9·7	23	1
1914	0·82	10·2	25	3

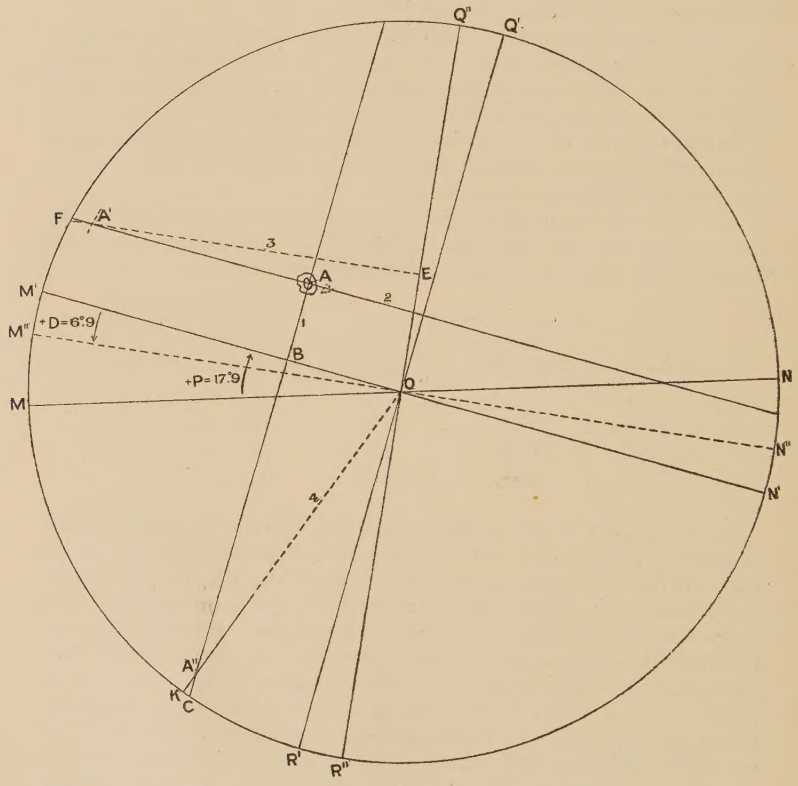
Two of the three greater magnetic storms of 1914, which occurred on April 5–6, accompanied the first transit of the Sun's disc of the area which, as described above, was intermittently disturbed from March 30 to November 15, and contained the large spot of August 21. They marked, too, the beginning of activity of a new solar cycle. Moderate magnetic disturbances were also probably connected with the same region on April 1, June 25, 26, 27, and November 11. The third greater magnetic storm of 1914, that of September 27, accompanied the transit of an active spot-group across the Sun's disc, September 24–October 2. This spot was situated upon almost exactly the same parallel of latitude as the spot-region discussed above, and removed from it just about 180° in longitude, that is, diametrically opposite to the former spot-region. As will be gathered from the table, the general revival of magnetic activity preceded that of sun-spot activity. It commenced in the year of absolute sun-spot minimum.

Stonyhurst College Observatory,
1915 *April* 6.

A Simple Geometrical Construction for determining the Heliographic Co-ordinates of Sun-spots. By F. Henroteau, D. ès Sc. (Brussels).

(*Communicated by the Rev. A. L. Cortie.*)

A photograph or drawing of the sun's disc with sun-spots having been obtained, with the E. and W. diameter marked upon



it, the heliographic co-ordinates of the spots can be measured, without the employment of micrometers, on projection discs in the following manner. Let P be the position-angle of the sun's axis, and D the heliographic latitude of the centre of the sun for the G.M.T. of observation, as determined by interpolation from the table in the *Companion to the Observatory* or from the *Nautical Almanac*. On a circle having the same diameter as the photograph or drawing, let MN be the diameter of the projected solar image. Draw M_IN_I, making with MN the angle P, and the diameter $M_{II}N_{II}$, making with M_IN_I the angle D. These angles are both

positive in the case illustrated, that of the large sun-spot of 1914 August 21. Draw also the diameters Q_IR_I, $Q_{II}R_{II}$, perpendicular respectively to M_IN_I and $M_{II}N_{II}$. The positions of these four lines are constant not only for one spot, but for any number of spots that it is required to measure on any given photograph or drawing. The construction for any spot, as, for example, the spot selected, requires four straight lines, as follows.

Heliographic Latitude.—Through A, the projected image of the spot, the lines 1 and 2 are drawn respectively perpendicular and parallel to M_IN_I. Line 1 meets M_IN_I in B and the circumference in C. With centre O and radius BC draw an arc of a circle cutting line 2 in A_I. Through this point A_I draw the line 3, parallel to $M_{II}N_{II}$. It meets $Q_{II}R_{II}$ in E and the circumference in F. The arc $M_{II}F$ measured on the circumference, which can be graduated in degrees if so desired, is the heliographic latitude of the spot. In the spot selected its value is $+18°{\cdot}9$.

Heliographic Longitude.—On the line 1 or BC make $BA_{II} = EA_I$. Join OA_{II} and produce it to meet the circumference in K, when R_IK will be the longitude of the spot relatively to the central meridian. By adding L, the heliographic longitude of the centre of the disc, as given in the table in the *Companion to the Observatory*, the necessary interpolation for the G.M.T. of observation having been made, the true heliographic longitude is obtained. Or the value of L can be put in on the figure, and longitudes measured directly. In the spot selected its value is $66°{\cdot}4$.

The construction therefore consists of four straight lines traced once for all for any given values of P and D, and four other straight lines for each spot to be measured on the same drawing. It has this advantage, that the straight lines 1, 2, 3 for different spots are parallel each to each, while the straight lines 4 all pass through the centre of the circumference. The point A_I, it must be noticed, is always placed to the left of the line Q_IR_I.

The principle of the figure lies in the combination in one of three projections of the sun, and the construction is based on the problem, given a projection of a point of the sphere on a plane, find the other projection on a plane perpendicular to the given plane. When the projections coincide, the problem may be solved by constructing only two straight lines and one arc.

The first projection of the spot A is made on a plane perpendicular to the line of sight, which is the image of the sun drawn or photographed. The second projection A_I is made on a solar meridian plane perpendicular to the plane of the first projection. It is obtained by the application of the problem stated above. The intersection of these two planes will therefore be R_IQ_I, perpendicular to M_IN_I, which makes the angle P with MN.

The solar axis of rotation being in the second plane of projection, must make an angle D with Q_IR_I. The projection of this axis will therefore be $Q_{II}R_{II}$. The perpendicular $M_{II}N_{II}$ is the projection of the solar equator, EF being the projection of a parallel of latitude passing through the spot, and the circumference being

the projection of a solar meridian. Hence $M_{II}F$ is the latitude of the spot.

$R_{II}Q_{II}$ in the second projection represents the meridian plane perpendicular to the plane of this projection. Let it be called α. A_IE is the distance of the spot from the plane α. Let us imagine that the second projection has turned through an angle D, clockwise, when R_IQ_I will then represent the plane α. Now, projecting the spot on this plane α is equivalent to projecting on the first plane of projection, after having turned the spot through the angle D about an axis perpendicular to the second plane of projection, which passes through the centre of the sun. The projection of the spot on the second plane α must therefore lie on the line AB.

If Q_IR_I is the projection of the plane α on the second plane of projection, it is also reciprocally the projection of the latter plane on α. Then M_IN_I will be the projection of the solar equatorial plane on α, and if we find a third projection of the spot, namely A_{II}, on the equatorial plane, it must lie on AB, and BA_{II} must be equal to A_IE, because BA_{II} is also the distance of the plane α from the spot. Joining OA_{II} and producing it to K, R_IK must be the longitude of the spot relatively to the central meridian.

To test the accuracy of this method of measurement a series of positions of spots was measured on the Stonyhurst drawings for 1886 July 28, 1886 October 3, and 1906 March 17, and compared with the Greenwich measures of the same spots on the same dates. Of twelve spots measured in latitude two positions were absolutely coincident, and the greatest deviation was $0°{\cdot}8$. The mean deviation was $0°{\cdot}4$ In longitude two positions were coincident, but the greatest deviation was $1°{\cdot}4$. The mean was $0°{\cdot}5$. On the average the method of measurement described will give correct positions within half a degree in latitude and longitude. It may be advantageously employed when no micrometers or discs are available.

Stonyhurst College Observatory:
1915 *March* 26.